Double Angle Identities:

$$\sin 2x = 2\sin x\cos x$$

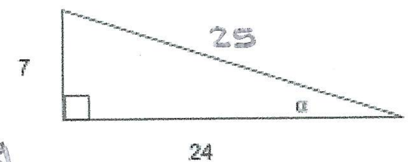
$$\cos 2x = \cos^2 x - \sin^2 x$$
$$= 2\cos^2 x - 1$$
$$= 1 - 2\sin^2 x$$

$$\tan 2x = \frac{2\tan x}{1-\tan^2 x}$$

1. Use the figure to find the exact value of:

$7^2 + 24^2 = h^2$

$25 = h$




a. $\sin 2\alpha = 2\sin\alpha\cos\alpha$

$= 2(7/25)(24/25)$

$= \boxed{\frac{336}{625}}$

b. $\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$

$= \left(\frac{24}{25}\right)^2 - \left(\frac{7}{25}\right)^2$

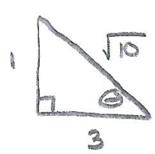
$= \boxed{\frac{527}{625}}$

*could use any of the 3 identities for $\cos 2\alpha$

c. $\tan 2\alpha = \frac{2\tan\alpha}{1-\tan^2\alpha}$

$= \frac{2(7/24)}{1-(7/24)^2} = \boxed{\frac{336}{527}}$

2. If $\cot\theta = 3$, and $\theta$ lies in quadrant III, find $\cos 2\theta$





$1^2 + 3^2 = h^2$

$\sqrt{10} = h$

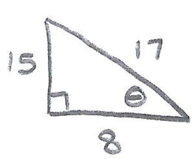
$\cos 2\theta = \cos^2\theta - \sin^2\theta$

$= (-3/\sqrt{10})^2 - (-1/\sqrt{10})^2$

$= \frac{8}{10}$

$= \boxed{4/5}$

3. If $\sin\theta = \frac{15}{17}$, and $\theta$ lies in quadrant II, find $\tan 2\theta$.





$15^2 + x^2 = 17^2$

$x = 8$

$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$

$= \frac{2(-15/8)}{1-(-15/8)^2}$

$= \frac{-30/8}{1-225/64}$

$= \frac{-30/8}{-161/64}$

$= \boxed{\frac{240}{161}}$

4. Write each expression as the sine, cosine, or tangent of a double angle. Then find the exact value of the expression.

a. $1 - 2\sin^2 \frac{\pi}{12}$ $= \cos(2 \cdot \pi/12)$

★ cos double angle identity

$= \cos \pi/6$

$= \boxed{\frac{\sqrt{3}}{2}}$

c. $2 \sin 22.5^\circ \cos 22.5^\circ$

$= \sin(2 \cdot 22.5^\circ)$

$= \sin(45^\circ)$

$= \boxed{\frac{\sqrt{2}}{2}}$

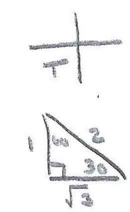
b. $\cos^2 105^\circ - \sin^2 105^\circ$

$= \cos(2 \cdot 105^\circ)$

$= \cos(210^\circ)$

$= \boxed{-\frac{\sqrt{3}}{2}}$

$\theta' = 30^\circ$




d. $\frac{2 \tan \frac{\pi}{8}}{1 - \tan^2 \frac{\pi}{8}}$ $= \tan(2 \cdot \pi/8)$

$= \tan \pi/4$

$= \boxed{1}$

5. Verify each identity:

a. $\sin 2\theta = \frac{2 \cot \theta}{1 + \cot^2 \theta}$

$2 \sin\theta \cos\theta =$

$= \frac{2(\cos\theta / \sin\theta)}{1 + (\cos\theta / \sin\theta)^2}$

$= \frac{2\cos\theta/\sin\theta}{1 + \frac{\cos^2\theta}{\sin^2\theta}}$

$= \frac{2\cos\theta/\sin\theta}{\frac{\sin^2\theta + \cos^2\theta}{\sin^2\theta}}$

$= \frac{2\cos\theta/\sin\theta}{1/\sin^2\theta} = \frac{2\cos\theta}{\sin\theta} \cdot \frac{\sin^2\theta}{1} = 2\cos\theta \sin\theta$ ✓

b. $(\sin \theta - \cos \theta)^2 = 1 - \sin 2\theta$

$= 1 - 2\sin\theta\cos\theta$

$\sin^2\theta - 2\sin\theta\cos\theta + \cos^2\theta =$

★ $\sin^2\theta + \cos^2\theta = 1$

$1 - 2\sin\theta\cos\theta =$ ✓

★ many different ways to prove these

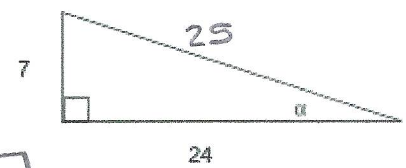
Half Angle Identities:

$$\sin\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}}$$

$$\cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}}$$

$$\tan\frac{\theta}{2} = \frac{\sin\theta}{1+\cos\theta} = \frac{1-\cos\theta}{\sin\theta}$$

6. Use the figure to find the exact value of:




a. $\sin\frac{\alpha}{2}$ $= \sqrt{\frac{1-\cos\alpha}{2}}$

$= \sqrt{\frac{1-(24/25)}{2}} = \sqrt{\frac{1/25}{2}} = \sqrt{1/50} = \frac{1}{5\sqrt{2}}$

$= \boxed{\frac{\sqrt{2}}{10}}$

b. $\cos\frac{\alpha}{2}$

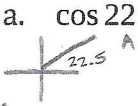
$= \sqrt{\frac{1+\cos\alpha}{2}} = \sqrt{\frac{1+24/25}{2}} = \sqrt{\frac{49/25}{2}} = \sqrt{\frac{49}{50}} = \frac{7}{5\sqrt{2}} = \boxed{\frac{7\sqrt{2}}{10}}$

c. $\tan\frac{\alpha}{2}$

$= \frac{\sin\alpha}{1+\cos\alpha} = \frac{7/25}{1+24/25} = \frac{7/25}{49/25} = \frac{7}{49} = \boxed{\frac{1}{7}}$

7. Use a half angle formula to find the exact value of each expression.

a. $\cos 22.5°$ $= \cos\left(45/2\right)$



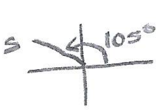

since 22.5° falls in quad I choose pos in front of √

$= +\sqrt{\frac{1+\cos 45}{2}}$

$= \sqrt{\frac{1+\frac{\sqrt{2}}{2}}{2}} = \sqrt{\frac{2+\sqrt{2}}{2}\cdot\frac{1}{2}} = \sqrt{\frac{2+\sqrt{2}}{4}}$

$= \boxed{\frac{\sqrt{2+\sqrt{2}}}{2}}$

b. $\sin 105°$




$= \sin\left(210°/2\right)$

$= +\sqrt{\frac{1-\cos 210}{2}}$

$= \sqrt{\frac{1-(-\sqrt{3}/2)}{2}}$

$= \sqrt{\frac{2+\sqrt{3}}{2}\cdot\frac{1}{2}}$

$= \sqrt{\frac{2+\sqrt{3}}{4}}$

$= \boxed{\frac{\sqrt{2+\sqrt{3}}}{2}}$

c. $\tan 112.5°$ $= \tan\left(225/2\right)$


$= \frac{\sin 225}{1+\cos 225}$

θ' = 45°

$= \frac{-\sqrt{2}/2}{1-\sqrt{2}/2}$

$= \frac{-\sqrt{2}/2}{\frac{2-\sqrt{2}}{2}} = \frac{-\sqrt{2}}{2}\cdot\frac{2}{2-\sqrt{2}}$

$= \frac{-\sqrt{2}}{2-\sqrt{2}}$

$= \frac{-\sqrt{2}(2+\sqrt{2})}{4-2}$

$= \frac{-2\sqrt{2}-2}{2}$

$\boxed{= -\sqrt{2} - 1}$

d. $\tan\frac{3\pi}{8}$

$= \tan\left(\frac{3\pi/4}{2}\right)$

$= \frac{1-\cos 3\pi/4}{\sin 3\pi/4}$ S +

$= \frac{1-(-\sqrt{2}/2)}{\sqrt{2}/2}$

$= \frac{2+\sqrt{2}}{2}\cdot\frac{2}{\sqrt{2}} = \frac{2+\sqrt{2}}{\sqrt{2}}$

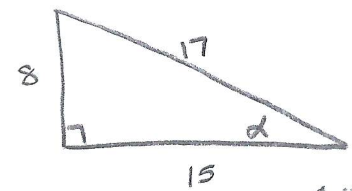
$= \frac{2\sqrt{2}+2}{2}$

$= \boxed{\sqrt{2}+1}$

$180° < \alpha < 270°$ $\quad 90° < \frac{\alpha}{2} < 135°$

8. $\tan\alpha = \frac{8}{15}$, $\alpha$ lies in quadrant III. Find:

S




$\alpha$ in quad 3

T

a. $\sin\frac{\alpha}{2}$

$$= +\sqrt{\frac{1-\cos\alpha}{2}}$$

$$= \sqrt{\frac{1-(-15/17)}{2}} = \sqrt{\frac{1+15/17}{2}} = \sqrt{\frac{32/17}{2}} = \sqrt{\frac{16}{17}} = \boxed{\frac{4\sqrt{17}}{17}}$$

b. $\cos\frac{\alpha}{2}$

★ $\alpha/2$ in quad 2 ★
so $\cos \alpha/2$ neg

$$= -\sqrt{\frac{1+\cos\alpha}{2}}$$

$$= -\sqrt{\frac{1+{-15}/17}{2}} = -\sqrt{\frac{2/17}{2}} = -\sqrt{1/17} = \boxed{-\frac{\sqrt{17}}{17}}$$

c. $\tan\frac{\alpha}{2}$

$$= \frac{1-\cos\alpha}{\sin\alpha}$$

$$= \frac{1-(-15/17)}{-8/17} = \frac{32/17}{-8/17} = -\frac{32}{8} = \boxed{-4}$$

9. Verify the identity: $\tan\frac{\alpha}{2} = \frac{\tan\alpha}{\sec\alpha+1}$

$$\frac{\sin\alpha}{1+\cos\alpha} =$$

$$= \frac{\frac{\sin\alpha}{\cos\alpha}}{\frac{1}{\cos\alpha}+1}$$

$$= \frac{\frac{\sin\alpha}{\cos\alpha}}{\frac{1+\cos\alpha}{\cos\alpha}}$$

$$= \frac{\sin\alpha}{\cos\alpha}\cdot\frac{\cos\alpha}{1+\cos\alpha}$$

$$= \frac{\sin\alpha}{1+\cos\alpha} \checkmark$$