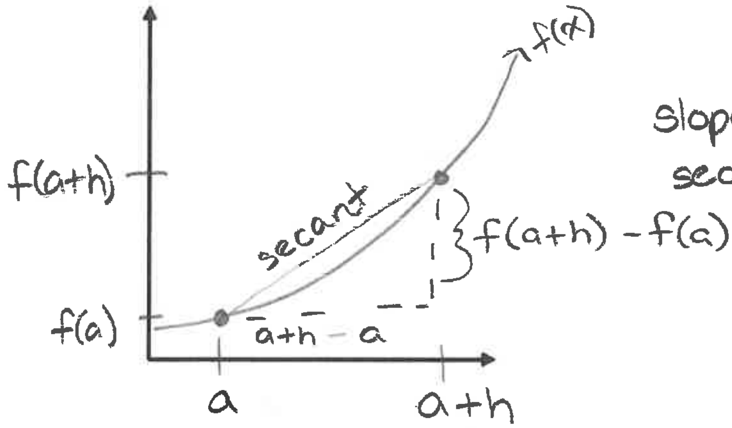
1. Find the average rate of change of $f(x) = \cos t$ over the interval $[0, \pi]$

$$\frac{\Delta y}{\Delta x} = \frac{f(\pi) - f(0)}{\pi - 0} = \frac{\cos\pi - \cos 0}{\pi} = \frac{-1-1}{\pi} = \frac{-2}{\pi}$$

Instantaneous Rate of Change



$$\text{slope of secant} = \frac{f(a+h) - f(a)}{a+h-a}$$

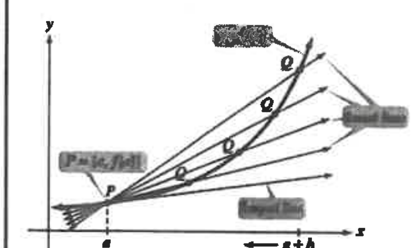
$$= \frac{f(a+h) - f(a)}{h}$$

Slope of the tangent Line to a Curve at a Point
The slope of the tangent line to the graph of a function $y = f(x)$ at $(a, f(a))$ is given by:

$$\lim_{h \to 0} \frac{f(a+h) - f(a)}{h} = \text{slope @ } a = f'(a)$$

Provided that this limit exists. This limit describes:
- The slope of the graph of f at $(a, f(a))$
- The instantaneous rate of change of f with respect to x at a

Want secant line to be as close to tangent line as possible:



Example:
1. Find the slope of the tangent line to the graph of $f(x) = x^2 + x$ at $(2, 6)$.

slope @ 2

$f(2) = 2^2 + 2 = 6$

$f(2+h) = (2+h)^2 + 2 + h$

$$\lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

$$\lim_{h \to 0} \frac{f(2+h) - f(2)}{h}$$

$$\lim_{h \to 0} \frac{(2+h)^2 + 2 + h - 6}{h}$$

$$\lim_{h \to 0} \frac{4 + 4h + h^2 + 2 + h - 6}{h}$$

$$\lim_{h \to 0} \frac{h^2 + 5h}{h}$$

$$\lim_{h \to 0} h + 5 = 5$$

2. Find the slope of the tangent line to the graph $f(x) = \frac{2}{x}$ at (1, 2). Then find the tangent line at the given point.

$$\lim_{h\to 0} \frac{f(1+h)-f(1)}{h}$$

$$\lim_{h\to 0} \frac{\frac{2}{1+h} - 2}{h}$$

$$\lim_{h\to 0} \frac{\frac{2}{1+h} - \frac{2(1+h)}{1+h}}{h}$$

$$\lim_{h\to 0} \frac{\frac{2}{1+h} - 2(1+h)}{\frac{h}{1}}$$

$$\lim_{h\to 0} \frac{2-2-2h}{1+h} \cdot \frac{1}{h}$$

$$\lim_{h\to 0} \frac{-2h}{h(1+h)}$$

$$\lim_{h\to 0} \frac{-2}{1+h} = -2$$

$$y-2=-2(x-1)$$

Limit Definition of the Slope (derivative)	Alternative Definition
$m_{tan} = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$ = slope at a	$m_{tan} = \lim_{x\to a} \frac{f(x)-f(a)}{x-a}$

3. The equations below, if evaluated, will give the slope of a tangent line at some exact x – value on a function. Determine what that x – value is and the function.

a. $\lim_{h\to 0} \frac{\sqrt{2+h}-\sqrt{2}}{h}$

$f(x) = \sqrt{x}$

$x = 2$

b. $\lim_{h\to 0} \frac{(1+h)^3-2-(-1)}{h}$

$f(x) = x^3 - 2$

$x = 1$

c. $\lim_{x\to 3} \frac{5x^2-45}{x-3}$

$f(x) = 5x^2$

$x = 3$

d. $\lim_{x\to \frac{1}{3}} \frac{\ln x - \ln(1/3)}{x-1/3}$

$f(x) = \ln x$

$x = 1/3$