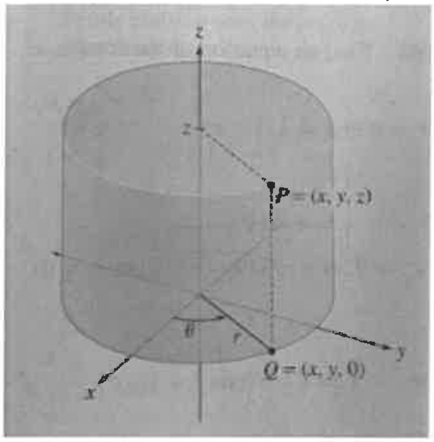
Cylindrical Coordinates (r, θ, z) * usually assume $r \geq 0$



Cylindrical to Rectangular	Rectangular to Cylindrical
$x = r\cos\theta$	$r = \sqrt{x^2+y^2}$
$y = r\sin\theta$	$\tan\theta = \frac{y}{x}$
$z = z$	$z = z$

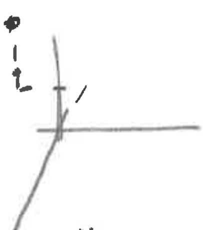
1. Find the rectangular coordinates of the point P with cylindrical coordinates $(r, \theta, z) = (2, \frac{3\pi}{4}, 5)$

$r = 2$ $\quad x = 2\cos 3\pi/4 \quad y = 2\sin 3\pi/4$

$\theta = 3\pi/4 \quad = 2(-1/\sqrt{2}) \quad = 2/\sqrt{2}$

$z = 5 \quad = -2/\sqrt{2}$

$\boxed{(-2/\sqrt{2}, 2/\sqrt{2}, 5)}$




2. Find cylindrical coordinates for the point with rectangular coordinates $(x, y, z) = (-3\sqrt{3}, -3, 5)$

$x = -3\sqrt{3} \quad \tan\theta = \frac{-3}{-3\sqrt{3}} \quad r = \sqrt{(-3\sqrt{3})^2 + (-3)^2}$

$= \sqrt{27+9}$

$y = -3 \quad \theta = \tan^{-1}(1/\sqrt{3}) \quad = \sqrt{36}$

$z = 5 \quad \theta = \pi/6 \quad \theta = 7\pi/6 \quad = 6$

$\boxed{(6, 7\pi/6, 5)}$

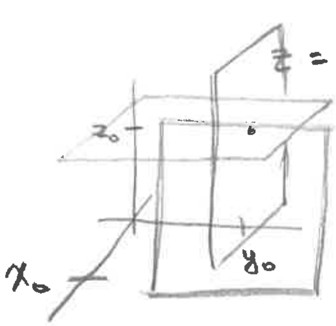
Level Surfaces: are the surfaces obtained by setting one of the coordinates equal to a constant

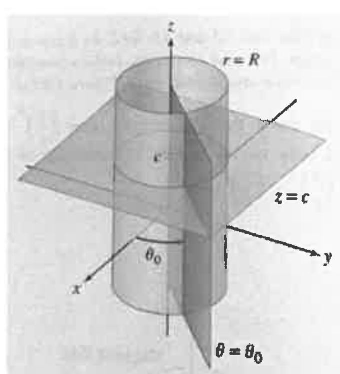
In rectangular coordinates:

$x = x_0$

$y = y_0$

$z = z_0$





In cylindrical coordinates:

$r = R$

$\theta = \theta_0$

$z = z$

3. Find an equation of the form $z = f(r, \theta)$ for the surfaces:

   a. $x^2 + y^2 + z^2 = 9$, with $z \geq 0$

   $r^2 + z^2 = 9$ or $z = \sqrt{9 - r^2}$

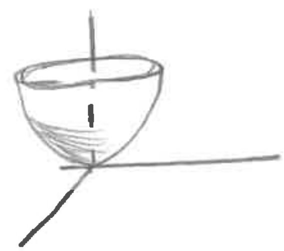
   b. $x + y + z = 1$

   $r\sin\theta + r\cos\theta + z = 1$

   $z = 1 - r(\sin\theta + \cos\theta)$

4. Graph the surface corresponding to the equation in cylindrical coordinates given by $z = r^2$

   $z = x^2 + y^2$

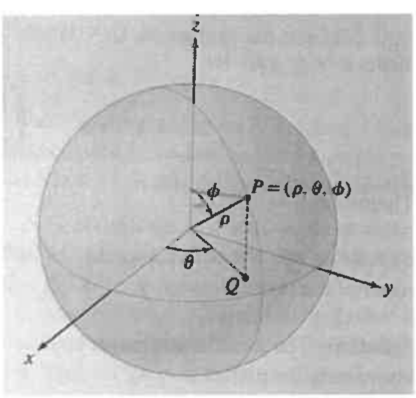


Spherical Coordinates (p, θ, ϕ)

can define a point P using two angles → θ and ϕ

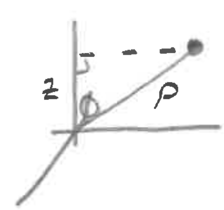
θ defines the angle on the xy-plane

ϕ defines the angle of declination → the angle between the z-axis and the ray through point P



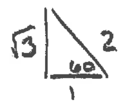
Restrict $p \geq 0$ and $0 \leq \phi \leq \pi$

Spherical to Rectangular	Rectangular to Spherical
$x = p\sin\phi\cos\theta$	*distance from origin $p = \sqrt{x^2 + y^2 + z^2}$
$y = p\sin\phi\sin\theta$	$\tan\theta = \frac{y}{x}$
$z = p\cos\phi$	$\cos\phi = \frac{z}{p}$



Find $r = p\sin\phi$ therefore:

$x = r\cos\theta = p\sin\phi\cos\theta$ $y = r\sin\theta = p\sin\phi\sin\theta$ $z = p\cos\phi$



5. Find the rectangular coordinates of $P = (p, \theta, \phi) = (3, \frac{\pi}{3}, \frac{\pi}{4})$, and find the radial coordinate r of its projection Q onto the xy −plane.

$p = 3$

$\theta = \pi/3$

$\phi = \pi/4$

$z = 3\cos \pi/4 = 3/\sqrt{2}$

$y = 3 \sin \pi/4 \sin \pi/3 = 3(1/\sqrt{2})(\sqrt{3}/2) = \frac{3\sqrt{3}}{2\sqrt{2}}$

$x = 3(1/\sqrt{2})\cos \pi/3 = 3(1/\sqrt{2})(1/2) = \frac{3}{2\sqrt{2}}$

$\boxed{(3/2\sqrt{2},\ 3\sqrt{3}/2\sqrt{2},\ 3/\sqrt{2})}$

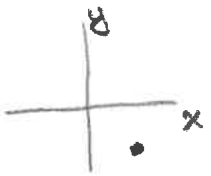
$r = \sqrt{(3/2\sqrt{2})^2 + (3\sqrt{3}/2\sqrt{2})^2} = \sqrt{9/8 + 27/8} = \sqrt{36/8}$ $\boxed{= \frac{3}{\sqrt{2}}}$

6. Find the spherical coordinates of the point $P = (x, y, z) = (2, -2\sqrt{3}, 3)$

$x = 2$

$y = -2\sqrt{3}$

$z = 3$



$\theta = \tan^{-1}(-2\sqrt{3}/2) = \tan^{-1}(-\sqrt{3}) = \frac{5\pi}{3}$

$p = \sqrt{(2)^2 + (-2\sqrt{3})^2 + 3^2} = \sqrt{4 + 12 + 9} = \sqrt{25} = 5$

$\phi = \cos^{-1}(3/5) \approx 0.93$

$\boxed{(5,\ 5\pi/3,\ 0.93)}$

7. Find an equation of the form $p = f(\theta, \phi)$ for the following surfaces:

a. $x^2 + y^2 + z^2 = 9$

$\boxed{p = 9}$

b. $z = x^2 - y^2$

$p\cos\phi = p^2 \sin^2\phi \cos^2\theta - p^2 \sin^2\phi \sin^2\theta$

$\cos\phi = p \sin^2\phi (\cos^2\theta - \sin^2\theta)$

$\cos\phi = p \sin^2\phi \cos 2\theta$

$\boxed{p = \frac{\cos\phi}{\sin^2\phi \cos 2\theta}}$

8. Graph $p = \sec\phi$

$p = \frac{p}{z}$

$\boxed{z = 1}$