Section 14.7: Maximum and Minimum values

Definition: A function of two variables has a **local maximum** at (a, b) if $f(x, y) \leq f(a, b)$ for all points (x, y) in some disk with center (a, b). The number $f(a, b)$ is called a **local maximum value.** If $f(x, y) \geq f(a, b)$ for all (x, y) in such disk, $f(a, b)$ is a **local minimum value.**

Note: The word local is sometimes replaced with the word relative.

Theorem: If f has a **local extremum** (that is, a local maximum or minimum) at (a, b) and the first-order partial derivatives of f exists there, then $f_x(a, b) = 0$ and $f_y(a, b) = 0$

Note: If the graph of f has a tangent plane at a local extremum, then the tangent plane is horizontal.

Definition: A point (a, b) such that $f_x(a, b) = 0$ and $f_y(a, b) = 0$, or one of these partial derivatives does not exist, is called a **critical point** of f.

Second Derivative Test: Suppose the second partial derivatives of f are continuous in a disk with center (a, b), and suppose that (a, b) is a critical point of f. Let

$$D = D(a, b) = f_{xx}(a, b) f_{yy}(a, b) - [f_{xy}(a, b)]^2$$

(a) If $D > 0$ and $f_{xx} > 0$, then $f(a, b)$ is a local minimum.
(b) If $D > 0$ and $f_{xx} < 0$, then $f(a, b)$ is a local maximum.
(c) If $D < 0$, then $f(a, b)$ is a sadle point.
(d) If $D = 0$ then the test gives no information.

Example: Find and classify the critical values of $f(x, y) = y^3 - 6y^2 - 2x^3 - 6x^2 + 48x + 20$

Example: Find and classify the critical values of $f(x, y) = x^3 + 6xy - 2y^2$

Example: Find and classify the critical values of $f(x, y) = 1 + 2xy - x^2 - y^2$