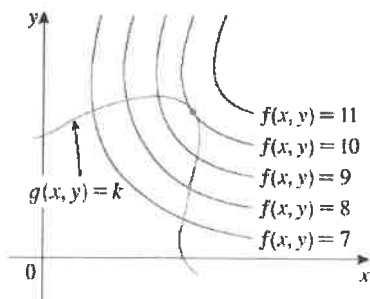## Section 14.8: Lagrange Multipliers

Lagrange multipliers are another method used to maximizing or minimizing a general function $f(x, y)$ subject to a constraint $g(x, y) = k$.

We are trying to find the extreme values of $f(x, y)$ subject to the condition $g(x, y) = k$. Thus we want to find the point(s) (a, b) on $g(x, y) = k$ so that the value of $f(a, b)$ will be a maximum or minimum.

Thus we want $f(x, y) = c$ some constant to just touch $g(x, y) = k$. This happens when both level curves have a common tangent line. This means that the normal lines where they touch, at the point (x_o, y_o) are identical. Thus

$\nabla f(x_o, y_o) = \lambda \nabla g(x_o, y_o)$ for some scalar λ.

The number λ is called a **Lagrange multiplier**.

Method of Lagrange multipliers: To find the maximum and minimum values of $f(x, y, z)$ subject to the constraints $g(x, y, z) = k$:

(a) Find all values of x, y, z, and λ such that

$\nabla f = \lambda \nabla g$ and $g(x, y, z) = k$

(b) Evaluate f at all points (x, y, z) that arise from step (a). The largest of these values is the maximum value of f and the smallest is the minimum value of f.

Note: If there are two constraints, g and h, then $\nabla f = \lambda \nabla g + \mu \nabla h$

Example: Find the extreme values of the function $f(x, y) = x^2 + 2y^2$ on the ellipse $x^2 + 16y^2 = 16$

Example: Find the points on the sphere $x^2 + y^2 + z^2 = 25$ where $f(x, y, z) = x + 2y + 3z$ has its maximum and minimum value.

Example: Find the point on the sphere $x^2 + y^2 + z^2 = 9$ that is closest to the point $(2, 3, 4)$.

Example: The base of a rectangular tank with volume of 540 cubic units is made of slate and the sides are made of glass. If slate costs five times as much as glass(per unit area), find the dimensions of the tank which minimize the cost of the materials.