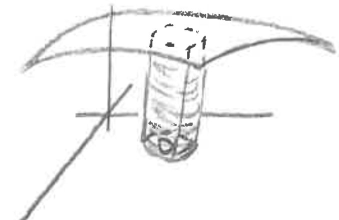
Scavenger Hunt of Textbook

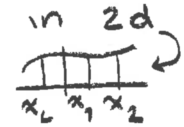
Goal is to practice reading a math textbook and understand concepts from that reading. I suggest reading page 885-887 alone, then complete the scavenger hunt below with groups.

1. Draw a picture of ~~area~~ volume under a ~~3d~~ surface



   a. How is this different from area under a curve in single variable calculus?

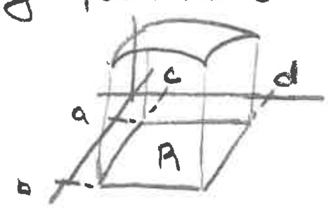
The domain is a plane in 3d vs an interval in 2d



The plane can be any region made up of curves & segments

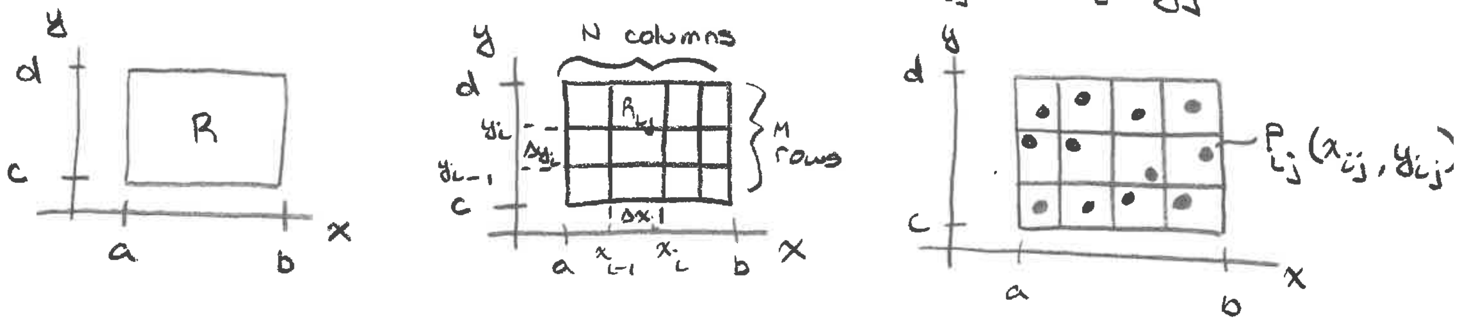
2. List the steps to defining a double integral

1) Subdivide $[a,b]$ and $[c,d]$ by choosing partitions
2) Create a grid of subrectangles (R_{ij})
3) Choose a sample point P_{ij} in each R_{ij}



* each rectangle has area $\Delta A_{ij} = \Delta x_i \Delta y_j$

3. Draw a picture of the process you defined above.



4. Write the Riemann sum of the area under a surface.

$$S_{N,M} = \sum_{i=1}^{N} \sum_{j=1}^{M} f(P_{ij}) \Delta A_{ij} = \sum_{i=1}^{N} \sum_{j=1}^{M} f(P_{ij}) \Delta x_i \Delta y_j$$

5. What is the volume of one cube under the surface?

$$f(P_{ij}) \Delta A_{ij} = f(P_{ij}) \Delta x_i \Delta y_j = \underbrace{(\text{height})(\text{area})}_{\text{volume of box}}$$

6. Describe the final step in defining the double integral.

$\|P\|$ = max width of the boxes subdividing volume

so want $\lim_{\|P\| \to 0}$

7. Write the definition of the Double Integral over a Rectangle:

$$\iint_R f(x,y)\,dA = \lim_{\|P\| \to 0} \sum_{i=1}^{N} \sum_{j=1}^{M} f(P_{ij})\,\Delta A_{ij}$$

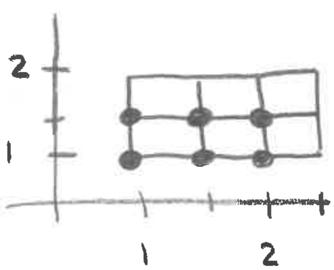
8. How do we partition x and y into regular subintervals?

$$\Delta x = \frac{b-a}{N} \qquad \Delta y = \frac{d-c}{M}$$

9. Example 1 (pg. 888): Let $R = [1, 2.5] \times [1, 2]$. Calculate $S_{3,2}$ for the integral $\iint_R xy\,dA$ of the graph of $z = xy$ using the following two choices of sample points:

a. Lower-left vertex

b. Midpoint of rectangle



$$\Delta x = \frac{2.5-1}{3} = \frac{1}{2} \qquad \Delta y = \frac{2-1}{2} = \frac{1}{2}$$

$$\Delta A = \Delta x\,\Delta y = \frac{1}{4}$$

$$S_{3,2} = \frac{1}{4} \sum_{i=1}^{3} \sum_{j=1}^{2} f(P_{ij})$$

a.

$$S_{3,2} = \frac{1}{4}\Big(f(1,1) + f(1, 3/2) + f(3/2, 1) + f(3/2, 3/2) + f(2, 1) + f(2, 3/2)\Big)$$

$$= \frac{1}{4}\left(1 + 3/2 + 3/2 + 9/4 + 2 + 3\right)$$

$$= 2.8125$$

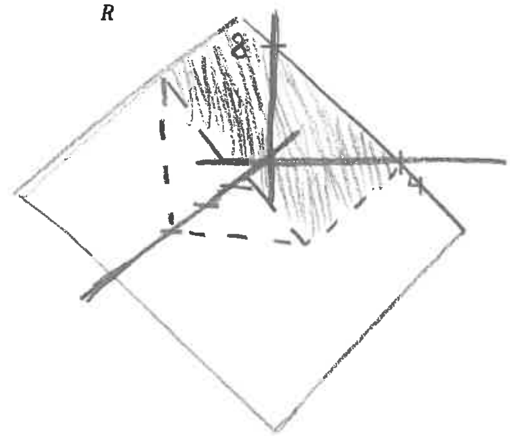
b.

$$S_{3,2} = \frac{1}{4}\Big(f(5/4, 5/4) + f(5/4, 7/4) + f(7/4, 5/4) + f(7/4, 7/4) + f(9/4, 5/4) + f(9/4, 7/4)\Big)$$

$$= \frac{1}{4}\left(25/16 + 35/16 + 35/16 + 49/16 + 45/16 + 63/16\right)$$

$$= 3.9375$$

10. Example 2 (pg. 888): Use geometry to evaluate $\iint_R (8-2y)\,dA$, where $R = [0,3] \times [0,4]$.



$\iint_R (8-2y)\,dA =$

$\frac{1}{2}(8)(4)(3)$

$= 48$

11. Theorem 1 states:

If a function f over 2 variables is continuous on a rectangle R, then f(x, y) is integrable over R

12. What are the two properties of double integrals:

Break up on + & -

1. $\left[\iint_R f(x,y) + g(x,y)\right] dA = \iint_R f(x,y)\,dA + \iint_R g(x,y)\,dA$

constant

2. $\iint_R c\,f(x,y)\,dA = c\iint_R f(x,y)\,dA$

It is a good practice to use other resources to help understand concepts after working with the textbook explanation. Some resources include:

- Youtube Video: Defining Double Integration with Riemann Sums | Volume under a Surface
  https://www.youtube.com/watch?v=JXh9AQkKmsw&list=PLHXZ9OQGMqxc_CvEy7xBKRQr6I214QJcd&index=25

- Pauls Online Notes: Double Integrals
  https://tutorial.math.lamar.edu/classes/calciii/DoubleIntegrals.aspx