1. Evaluate the following:

*evaluate inner integral holding outer variable constant

a. $\int_2^4\left(\int_1^9 ye^x dy\right)dx$

*x constant

$= \int_2^4 \left[\frac{1}{2}y^2 e^x \Big|_1^9\right] dx$

$= \int_2^4 \left[\frac{1}{2}(81-1)e^x\right] dx$

$= \int_2^4 40e^x \, dx$

$= 40e^x \Big|_2^4 = \boxed{40e^4 - 40e^2}$

b. $\int_{y=0}^{4}\int_{x=0}^{3} \frac{dx\,dy}{\sqrt{3x+4y}}$

ex) dx dy

y held constant first

*y constant

$\int_0^4 \left(\int_0^3 (3x+4y)^{-1/2} dx\right) dy$

$u = 3x + 4y$

$du = 3\,dx$

$\frac{1}{3}du = dx$

$\int_0^4 \left[\int_0^3 \frac{1}{3}(u)^{-1/2} du\right] dy$

$\int_0^4 \left[\frac{2}{3}u^{1/2} \Big|_{0+4y}^{9+4y}\right] dy$

$\int_0^4 \frac{2}{3}\left(\sqrt{9+4y} - \sqrt{4y}\right) dy$

$\frac{2}{3}\int_0^4 (9+4y)^{1/2} dy - \frac{2}{3}\int_0^4 (4y)^{1/2} dy$

$u = 9+4y$

$du = 4\,dy$

$\frac{1}{4}du = dy$

$\frac{2}{3}\cdot\frac{1}{4}\int_9^{25} u^{1/2} du - \frac{2}{3}\cdot\frac{1}{4}\int_0^{16} u^{1/2} du$

$\frac{1}{6}\,\frac{2}{3}u^{3/2}\Big|_9^{25} - \frac{1}{6}\,\frac{2}{3}u^{3/2}\Big|_0^{16}$

$\frac{1}{9}\left[\sqrt{25}^3 - \sqrt{9}^3 - \left(\sqrt{16}^3 - 0\right)\right]$

$\frac{1}{9}(125 - 27 - 64)$

$\boxed{34/9}$

2. Verify that $\int_{y=0}^{4}\int_{x=0}^{3} \frac{dx\,dy}{\sqrt{3x+4y}} = \int_{x=0}^{3}\int_{y=0}^{4} \frac{dx\,dy}{\sqrt{3x+4y}}$

3. Find the volume between the graph of $f(x, y) = 16 - x^2 - 3y^2$ and the rectangle $R = [0, 3] \times [0, 1]$

$$\int_0^3\int_0^1 (16-x^2-3y^2)\,dy\,dx$$

$$=\int_0^3 \left[16y-x^2y-y^3\Big|_0^1\right]dx$$

$$=\int_0^3 [(16-x^2-1)-(0-0-0)]\,dx$$

$$=\int_0^3 15-x^2\,dx$$

$$=15x-\frac{1}{3}x^3\Big|_0^3$$

$$=(45-9)-(0)$$

$$=\boxed{36}$$

4. Calculate $\iint_R \frac{dA}{(x+y)^2}$, where $R = [1, 2] \times [0, 1]$

$$\int_1^2\int_0^1 (x+y)^{-2}\,dy\,dx$$

$$=\int_1^2 \left[-(x+y)^{-1}\Big|_0^1\right]dx$$

$$=\int_1^2 -(x+1)^{-1}+x^{-1}\,dx$$

$$=-\int_1^2 \frac{1}{x+1}\,dx+\int_1^2 \frac{1}{x}\,dx$$

$$=-\ln|x+1|+\ln|x|\Big|_1^2$$

$$=\ln\frac{x}{x+1}\Big|_1^2$$

$$=\ln\frac{2}{3}-\ln\frac{1}{2}=\ln\frac{2/3}{1/2}$$

$$=\boxed{\ln 4/3}$$

5. Evaluate the following:

a. $\int_1^3\int_0^2 x^3y\,dy\,dx$

$$\int_1^3\left[\frac{1}{2}x^3y^2\Big|_0^2\right]dx$$

$$=\int_1^3 \left(\frac{1}{2}x^3(4)-0\right)dx$$

$$=\int_1^3 2x^3\,dx$$

$$=\frac{2x^4}{4}\Big|_1^3=\frac{1}{2}(81)-\frac{1}{2}$$

$$=\boxed{40}$$

b. $\int_{-1}^1\int_0^\pi x^2\sin y\,dy\,dx$

$$\int_{-1}^1\left[-x^2\cos y\Big|_0^\pi\right]dx$$

$$\int_{-1}^1 (-x^2\cos\pi+x^2\cos 0)\,dx$$

$$\int_{-1}^1 (x^2+x^2)\,dx$$

$$\int_{-1}^1 2x^2\,dx=\frac{2}{3}x^3\Big|_{-1}^1$$

$$=2/3+2/3=\boxed{4/3}$$

c. $\int_0^{\pi/4}\int_{\pi/4}^{\pi/2} \cos(2x + y)\,dy\,dx$

$$\int_0^{\pi/4} \sin(2x+y) \Big|_{\pi/4}^{\pi/2} dx$$

$$\int_0^{\pi/4} \sin(2x+\pi/2) - \sin(2x+\pi/4)\, dx$$

$$-\tfrac{1}{2}\cos(2x+\pi/2) + \tfrac{1}{2}\cos(2x+\pi/4)\Big|_0^{\pi/4}$$

$$\left[-\tfrac{1}{2}\cos(\pi/2+\pi/2) + \tfrac{1}{2}\cos(\pi/2+\pi/4)\right] - \left[-\tfrac{1}{2}\cos\pi/2 + \tfrac{1}{2}\cos\pi/4\right]$$

$$= \tfrac{1}{2} - \sqrt{2}/4 + 0 - \frac{\sqrt{2}}{4} = \boxed{\frac{2-2\sqrt{2}}{4}} \approx -0.207$$

d. $\int_1^2\int_2^4 e^{3x-y}\,dy\,dx$

$$\int_1^2 -e^{3x-y}\Big|_2^4 dx$$

$$\int_1^2 -e^{3x-4} + e^{3x-2}\, dx$$

$$-\tfrac{1}{3}e^{3x-4} + \tfrac{1}{3}e^{3x-2}\Big|_1^2$$

$$\boxed{-\tfrac{1}{3}e^{2} + \tfrac{1}{3}e^{4} + \tfrac{1}{3}e^{-1} - \tfrac{1}{3}e^{1}}$$

≈ 14.953

e. $\int_0^8\int_1^2 \frac{x\,dx\,dy}{\sqrt{x^2+y}}$

$u = x^2 + y$

$\tfrac{1}{2}du = x\,dx$

$$\tfrac{1}{2}\int_0^8 u^{-1/2}\, du\, dy$$

$$\int_0^8 \tfrac{1}{2}\, 2\, u^{1/2}\Big|_1^2 dy$$

$$\int_0^8 (x^2+y)^{1/2}\Big|_1^2 dy$$

$$\int_0^8 (4+y)^{1/2} - (1+y)^{1/2}\, dy$$

$$\tfrac{2}{3}(4+y)^{3/2} - \tfrac{2}{3}(1+y)^{3/2}\Big|_0^8$$

$$\boxed{\tfrac{2}{3}\left(12^{3/2} - 9^{3/2} - 4^{3/2} + 1^{3/2}\right)}$$

f. $\int_1^2\int_1^3 \frac{\ln(xy)\,dy\,dx}{y}$

$u = \ln(xy)$

$du = x/xy\; dy$

$= \frac{1}{y}\,dy$

$$\int_1^2\int_{\ln x}^{\ln 3x} u\, du$$

$$\int_1^2 \tfrac{1}{2}u^2\Big|_{\ln x}^{\ln 3x} dx$$

$$\tfrac{1}{2}\int_1^2 (\ln 3x)^2 - (\ln x)^2\, dx$$

* by parts *

see back

$$\frac{1}{2}\left[\int_1^2 (\ln 3x)^2\,dx - \int_1^2 (\ln x)^2\,dx\right]$$

$$u = (\ln 3x)^2 \qquad v = x \qquad\qquad u = (\ln x)^2 \qquad v = x$$

$$du = \frac{2\ln x}{x} \qquad dv = dx$$

$$du = \frac{2\ln 3x}{3x}\cdot 3 \qquad dv = dx$$

$$= \frac{2\ln 3x}{x}$$

$$\frac{1}{2}\left[x(\ln 3x)^2\Big|_1^2 - \int_1^2 2\ln 3x\,dx - \left(x(\ln x)^2\Big|_1^2 - \int_1^2 2\ln x\,dx\right)\right]$$

$$u = \ln 3x \qquad v = x \qquad\qquad u = \ln x \qquad v = x$$

$$du = \frac{1}{x} \qquad dv = dx \qquad\qquad du = \frac{1}{x} \qquad dv = dx$$

$$\frac{1}{2}\left[x(\ln 3x)^2\Big|_1^2 - 2\left(x\ln 3x\Big|_1^2 - \int_1^2 1\,dx\right) - \left(x(\ln x)^2\Big|_1^2 - 2\left(x\ln x\Big|_1^2 - \int_1^2 1\,dx\right)\right)\right]$$

$$\frac{1}{2}\left[x(\ln 3x)^2 - 2x\ln 3x + 2x - x(\ln x)^2 + 2x\ln x - 2x\Big|_1^2\right]$$

$$\frac{1}{2}\left[2(\ln 6)^2 - 4\ln 6 + \cancel{4} - 2(\ln 2)^2 + 4\ln 2 - \cancel{4} - \left((\ln 3)^2 - 2\ln 3 + \cancel{2}\right.\right.$$

$$\left.\left. - \cancel{(\ln 1)^2} + \cancel{2\ln 1} - \cancel{2}\right)\right]$$

$$\frac{1}{2}\left[2(\ln 6)^2 - 4\ln 6 + 4\ln 2 - (\ln 3)^2 + 2\ln 3\right]$$

$$(\ln 6)^2 - 2\ln 6 + 2\ln 2 - \frac{1}{2}(\ln 3)^2 + \ln 3$$

$$(\ln 6)^2 - \frac{1}{2}(\ln 3)^2 - \ln 36 + \ln 4 + \ln 3$$

$$(\ln 6)^2 - \tfrac{1}{2}(\ln 3)^2 + \ln \tfrac{12}{36} = (\ln 6)^2 - \tfrac{1}{2}(\ln 3)^2 + \ln \tfrac{1}{3}$$