Grade

2018 FRQ, #6, 13, 15
part c

E D E

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Name

confirms r<1 so can find sum

1. If b and t are real numbers such that $0 < |t| < |b|$, which of the following infinite series has sum $\frac{1}{b^2+t^2}$?

$\frac{a_1}{1-r}$ ★ can ① check all a-d individually or ② manipulate $\frac{1}{b^2+t^2}$ to be in proper form $\frac{a_1}{1-r}$

(A) $\frac{1}{b^2}\sum_{k=0}^{\infty}\left(\frac{t^2}{b^2}\right)^k$

(B) $\frac{1}{b^2}\sum_{k=0}^{\infty}(-1)^k\left(\frac{t^2}{b^2}\right)^k$

(C) $b^2\sum_{k=0}^{\infty}\left(\frac{t^2}{b^2}\right)^k$

(D) $b^2\sum_{k=0}^{\infty}(-1)^k\left(\frac{t^2}{b^2}\right)^k$

option 1

b) $a_1 = 1/b^2$

$r = -t^2/b^2$

$\frac{1/b^2}{1+t^2/b^2} = \frac{1}{b^2+t^2}$

option 2

$\frac{1}{b^2(1+t^2/b^2)} = \frac{1/b^2}{1+t^2/b^2}$

$a_1 = 1/b^2$

$r = -t^2/b^2$

so B

★ recognize plus must have resulted w/ r being negative so only check b and d

2.

x	$g(x)$	$g'(x)$	$g''(x)$	$g'''(x)$	$g^{(4)}(x)$
-3	1	-2	-4	2	16

Selected values of a function g and its first four derivatives are shown in the table above. What is the approximation for the value of $g(-2)$ obtained by using the third-degree Taylor polynomial for g about $x = -3$?

$T_3(x) = 1 - 2(x+3) - \frac{4}{2!}(x+3)^2 + \frac{2}{3!}(x+3)^3$

(A) $-\frac{8}{3}$

(B) $-\frac{7}{3}$

(C) -2

(D) -3

$g(-2) \approx T_3(-2) = 1 - 2(-2+3) - 2(-2+3)^2 + \frac{2(-2+3)^3}{6}$

$= 1 - 2 - 2 + 1/3$

$= -8/3$

3. Let $T_3(x)$ be the third-degree Taylor polynomial for $f(x) = x^3$ about $x = 2$. Which of the following statements is true?

$f'(x) = 3x^2$

$f'(2) = 12$

$f''(x) = 6x$

$f''(2) = 12$

$f^{(3)}(x) = 6$

$f^{(3)}(2) = 6$

$T_3(x) = 8 + 12(x-2) + \frac{12}{2!}(x-2)^2 + \frac{6(x-2)^3}{3!}$

$= 8 + 12(x-2) + 6(x-2)^2 + (x-2)^3$

all other derivatives = 0 so $T_3(x) = f(x) = x^3$
so all other terms in taylor series are 0

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(A) $T_3(x) = 8 + 12(x-2) + 12(x-2)^2 + 6(x-2)^3$, and $T_3(x)$ provides a good approximation for $f(x)$ only for values of x that are close to $x = 2$.

(B) $T_3(x) = 8 + 12(x-2) + 12(x-2)^2 + 6(x-2)^3$, and $T_3(x)$ provides a good approximation for $f(x)$ for all real numbers x.

(C) $T_3(x) = 8 + 12(x-2) + 6(x-2)^2 + (x-2)^3$, and $T_3(x)$ provides a good approximation for $f(x)$ only for values of x that are close to $x = 2$.

(D) $T_3(x) = 8 + 12(x-2) + 6(x-2)^2 + (x-2)^3$, and $T_3(x)$ provides a good approximation for $f(x)$ for all real numbers x.

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4. Let f be the function defined by $f(x) = \sqrt{x}$. What is the approximation for the value of $\sqrt{3}$ obtained by using the second-degree Taylor polynomial for f about $x = 4$?

(A) $\frac{55}{32}$

(B) $\frac{111}{64}$

(C) $\frac{143}{64}$

(D) $\frac{167}{64}$

$f'(x) = \frac{1}{2}x^{-1/2}$ $\quad f'(4) = \frac{1}{2\sqrt{4}} = \frac{1}{4}$

$f''(x) = \frac{-1}{4}x^{-3/2}$ $\quad f''(4) = \frac{-1}{4\sqrt{4^3}} = \frac{-1}{4\cdot 8} = \frac{-1}{32}$

$P_2(x) = 2 + \frac{1}{4}(x-4) - \frac{\frac{1}{32}(x-4)^2}{2!}$

$= 2 + \frac{1}{4}(x-4) - \frac{1}{64}(x-4)^2$

$P_2(3) = 2 + \frac{1}{4}(3-4) - \frac{1}{64}(3-4)^2$

$= 2 - \frac{1}{4} - \frac{1}{64}$

$= \frac{128}{64} - \frac{16}{64} - \frac{1}{64}$

$= \frac{111}{64}$

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5. What is the value of $\sum_{n=0}^{\infty}\left(-\frac{2}{3}\right)^n$?

$r = -\frac{2}{3}$ $\quad a_1 = 1$

$\frac{1}{1+\frac{2}{3}} = \frac{1}{\frac{5}{3}} = \frac{3}{5}$

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(A) −2

(B) $-\frac{2}{5}$

(C) $\frac{3}{5}$

(D) 3

(E) The series diverges.

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6. Which of the following series converge to 2?

I. $\sum_{n=1}^{\infty} \frac{2n}{n+3}$ $\xrightarrow{\text{partial sums}}$ $s_1 = \frac{2}{4} = 0.5$

$s_2 = \frac{2}{4} + \frac{4}{5} = 1.3$

geo ✓II. $\sum_{n=1}^{\infty} \frac{-8}{(-3)^n}$

$s_3 = \frac{2}{4} + \frac{4}{5} + \frac{6}{6} = 2.3$

geo ✓III. $\sum_{n=0}^{\infty} \frac{1}{2^n}$

$s_4 = \frac{2}{4} + \frac{4}{5} + \frac{6}{6} + \frac{8}{7} = 3.44$

doesnt converge to 2

(A) I only

(B) II only

(C) III only

(D) I and III only

(E) II and III only

$r = -\frac{1}{3}$ $a_1 = 8/3$

$\frac{8/3}{1 + 1/3} = 2$ ✓

$r = 1/2$ $a_1 = 1$

$\frac{1}{1 - 1/2} = \frac{1}{1/2} = 2$ ✓

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calc 7. If $f(x) = \sum_{k=1}^{\infty} (\sin^2 x)^k$, then $f(1)$ is

geo

$r = \sin^2 x$ $a_1 = \sin^2 x$

$f(x) = \frac{\sin^2 x}{1 - \sin^2 x}$

$f(1) = \frac{\sin^2(1)}{1 - \sin^2(1)} = 2.4255$

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(A) 0.369

(B) 0.585

(C) 2.400

(D) 2.426

(E) 3.426

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8. To what number does the series $\sum_{k=0}^{\infty}\left(\frac{-e}{\pi}\right)^k$ converge?

geo $r = -e/\pi$ $a_1 = 1$

$\frac{1}{1+e/\pi} = \frac{1}{\frac{\pi+e}{\pi}} = \frac{\pi}{\pi+e}$

(A) 0

(B) $\frac{-e}{\pi+e}$

(C) $\frac{\pi}{\pi+e}$

(D) The series does not converge.

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9. Consider the series $\sum_{n=1}^{\infty} a_n$. If a_1 = 16 and $\underbrace{(a_{n+1}/a_n) = 1/2}_{r = 1/2 \text{ geo}}$ for all integers n ≥ 1, then $\sum_{n=1}^{\infty} a_n$ is

$\frac{16}{1-1/2} = \frac{16}{1/2} = 32$

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(A) 0

(B) 2

(C) 17

(D) 32

(E) divergent

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calc

10. Let f be a function with $f(3) = 2$, and $f'(3) = -1$, $f''(3) = 6$, and $f'''(3) = 12$. Which of the following is the third-degree Taylor polynomial for f about $x = 3$?

(A) $2 - (x - 3) + 3(x - 3)^2 + 2(x - 3)^3$

$P_3(x) = 2 - 1(x-3) + \frac{6(x-3)^2}{2!} + \frac{12}{3!}(x-3)^3$

(B) $2 - (x - 3) + 3(x - 3)^2 + 4(x - 3)^3$

(C) $2 - (x - 3) + 6(x - 3)^2 + 12(x - 3)^3$

$= 2 - (x-3) + 3(x-3)^2 + 2(x-3)^3$

(D) $2 - x + 3x^2 + 2x^3$

(E) $2 - x + 6x^2 + 12x^3$

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11. Let $P(x) = 3 - 3x^2 + 6x^4$ be the fourth-degree Taylor polynomial for the function f about $x = 0$. What is the value of $f^{(4)}(0)$?

$\frac{f^{(4)}(0)\,x^4}{4!} = 6x^4$

$\frac{f^{(4)}(0)}{4!} = 6 \qquad f^{(4)}(0) = 6 \cdot 24$

$= 144$

(A) 0

(B) $\frac{1}{4}$

(C) 6

(D) 24

(E) 144

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calc

12. Let $P(x) = 3x^2 - 5x^3 + 7x^4 + 3x^5$ be the fifth-degree Taylor polynomial for the function f about $x = 0$. What is the value of $f'''(0)$?

(A) -30

(B) -15

(C) -5

(D) $-\frac{5}{6}$

(E) $-\frac{1}{6}$

$$\frac{f'''(0)x^3}{3!} = -5x^3$$

$$\frac{f^{(3)}(0)}{3!} = -5$$

$$f^{(3)}(0) = -5(6)$$

$$= -30$$

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13. The third-degree Taylor polynomial for a function f about $x = 4$ is $\frac{(x-4)^3}{512} - \frac{(x-4)^2}{64} + \frac{(x-4)}{4} + 2$. What is the value of $f'''(4)$?

$$\frac{1}{512} = \frac{f^{(3)}(4)}{3!}$$

$$\frac{6}{512} = f^{(3)}(4)$$

$$3/256 =$$

(A) $-\frac{1}{64}$

(B) $-\frac{1}{32}$

(C) $\frac{1}{512}$

(D) $\frac{3}{256}$

(E) $\frac{81}{256}$

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calc **14.** The nth derivative of a function f at $x=0$ is given by $f^{(n)}(0) = (-1)^n \frac{n+1}{(n+2)2^n}$ for all $n \geq 0$. Which of the following is the Maclaurin series for f ?

(A) $-\frac{1}{2} + \frac{1}{3}x - \frac{3}{32}x^2 + \frac{1}{60}x^3 - ...$

(B) $\frac{1}{2} - \frac{1}{3}x + \frac{3}{16}x^2 + \frac{1}{10}x^3 + ...$

(C) $\frac{1}{2} + \frac{1}{3}x + \frac{3}{32}x^2 + \frac{1}{60}x^3 + ...$

(D) $\frac{1}{2} - \frac{1}{3}x + \frac{3}{32}x^2 - \frac{1}{60}x^3 + ...$

(E) $\frac{1}{2} - 3x + \frac{32}{3}x^2 - 60x^3 + ...$

$f^{(0)}(0) = \frac{1}{2}$

$f^{(1)}(0) = \frac{-2}{3 \cdot 2} = -\frac{1}{3}$

$f^{(2)}(0) = \frac{3}{4 \cdot 2^2} = \frac{3}{16}$

$\frac{1}{2} - \frac{1}{3}x + \frac{\frac{3}{16}}{2!}x^2$

$\frac{1}{2} - \frac{1}{3}x + \frac{3}{32}x^2$

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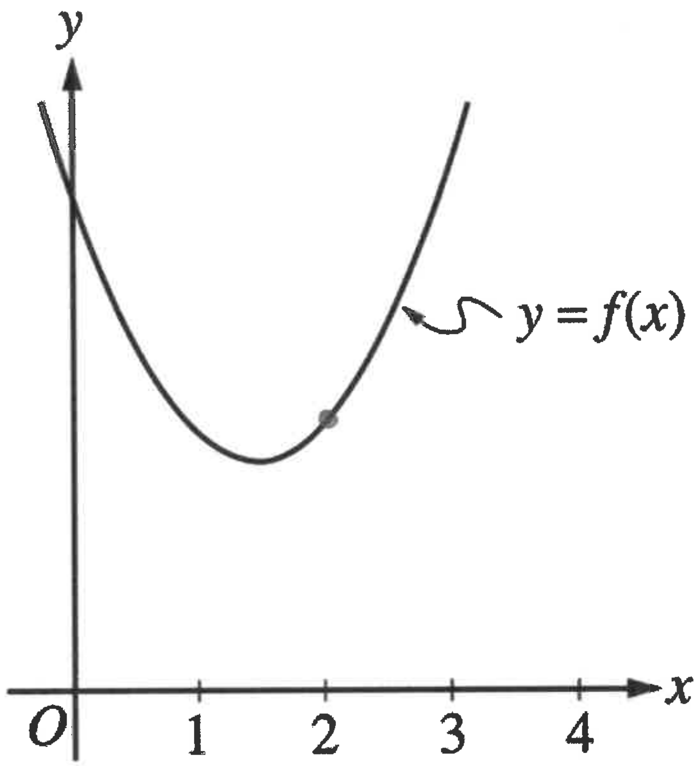
15.



The figure above shows the graph of a function f. Which of the following could be the second-degree Taylor polynomial for f about x = 2 ?

(A) $2 - x - x^2$

(B) $2 + x - x^2$

(C) $2 - (x-2) + (x-2)^2$

(D) $2 + (x-2) - (x-2)^2$

(E) $2 + (x-2) + (x-2)^2$

$f(2) + f'(2)(x-2) + \frac{f''(2)}{2!}(x-2)^2$

$f(2) > 0$ $f'(2) > 0$ $f''(2) > 0$

b/c inc b/c concave up

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calc 16. Let f be a function with $f(0) = 1$, $f'(0) = 2$, and $f''(0) = -2$. Which of the following could be the graph of the second-degree Taylor polynomial for f about $x = 0$?

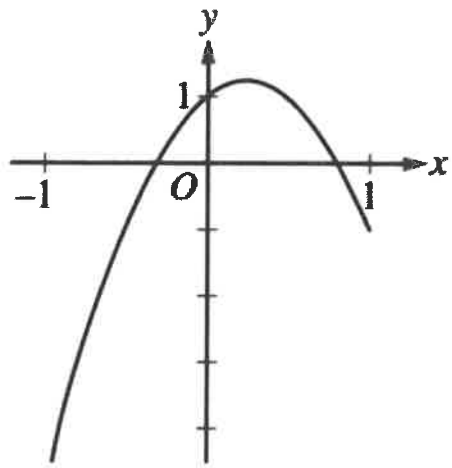
increasing @ $x=0$

concave down @ $x=0$

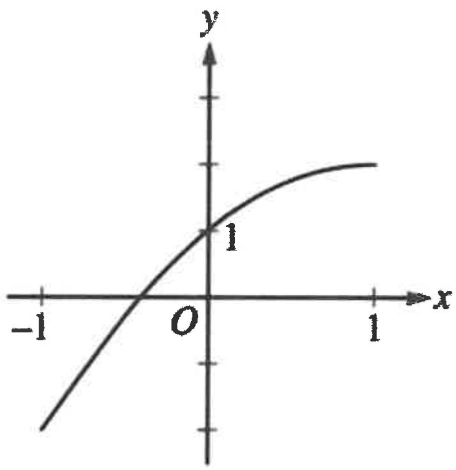
narrows to a and b

* remember graph is of $P_2(x)$ not $f(x)$ so $P_2(x)$ always inc and always concave down b/c no additional terms
↳ stops @ $\frac{f''(0)x^2}{2!}$

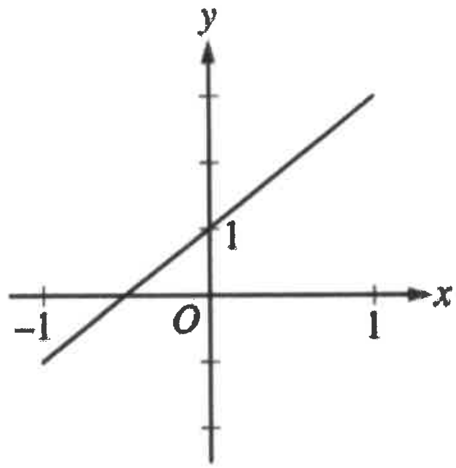
(A)



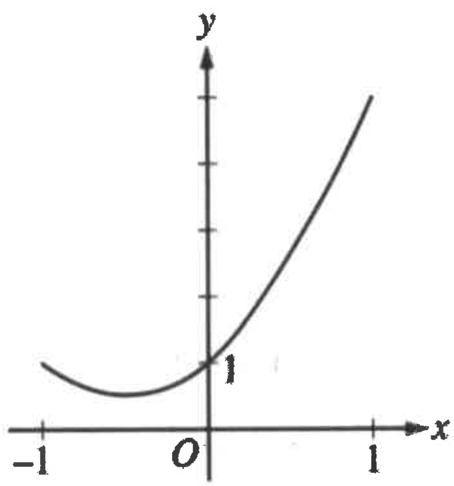
(B)



(C)



(D)



calc **17.** Let f be a function having derivatives of all orders for $x > 0$ such that $f(3) = 2$, $f'(3) = -1$, $f''(3) = 6$, and $f'''(3) = 12$. Which of the following is the third-degree Taylor polynomial for f about $x = 3$?

$$2 - 1(x-3) + \frac{6}{2!}(x-3)^2 + \frac{12}{3!}(x-3)^3$$

$$2 - (x-3) + 3(x-3)^2 + 2(x-3)^3$$

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(A) $2 - x + 6x^2 + 12x^3$

(B) $2 - x + 3x^2 + 2x^3$

(C) $2 - (x-3) + 6(x-3)^2 + 12(x-3)^3$

(D) $2 - (x-3) + 3(x-3)^2 + 4(x-3)^3$

(E) $2 - (x-3) + 3(x-3)^2 + 2(x-3)^3$

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calc **18.** The function f has derivatives of all orders for all real numbers with *f(0) = 3, f'(0) = -4, f''(0) = 2*, and *f'''(0) = 1*. Let *g* be the function given by $g(x) = \int_0^x f(t)\,dt$. What is the third-degree Taylor polynomial for *g* about *x = 0* ?

(A) $-4x + 2x^2 + \frac{1}{3}x^3$

(B) $-4x + x^2 + \frac{1}{6}x^3$

(C) $3x - 2x^2 + \frac{1}{3}x^3$

(D) $3x - 2x^2 + \frac{2}{3}x^3$

(E) $3 - 4x + x^2 + \frac{1}{6}x^3$

$g(0) = 0$

$g'(x) = f(x) \quad g'(0) = f(0) = 3$

$g''(x) = f'(x) \quad g''(0) = f'(0) = -4$

$g'''(x) = f''(x) \quad g'''(0) = f''(0) = 2$

$0 + 3(x-0) - \frac{4}{2!}x^2 + \frac{2}{3!}x^3$

$3x - 2x^2 + \frac{1}{3}x^3$

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19. What is the radius of convergence of the Maclaurin series for $\frac{2x}{1+x^2}$?

$r = -x^2$

$|-x^2| < 1$

$x^2 < 1$

$|x| < 1$

radius = 1

$x < 1$ and $x > -1$

$-1 < x < 1$

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(A) 1/2

(B) 1

(C) 2

(D) infinite

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20. What is the radius of convergence for the power series $\sum_{n=0}^{\infty} \frac{(x-4)^n}{2 \cdot 3^{n+1}}$? $= \sum \frac{(x-4)^n}{2 \cdot 3 \cdot 3^n}$

(A) $\frac{1}{3}$

(B) $\frac{3}{2}$

(C) 3

(D) 4

(E) 6

$a_1 = \frac{1}{6}$ $\quad r = \frac{(x-4)}{3}$

$-1 < \frac{x-4}{3} < 1$

$-3 < x-4 < 3$

$1 < x < 7$ centered @ $x=4$

so $R = 3$

Also can think of it as:

$\left|\frac{x-4}{3}\right| < 1$

$\frac{|x-4|}{3} < 1$

$|x-4| < 3$ radius

centered

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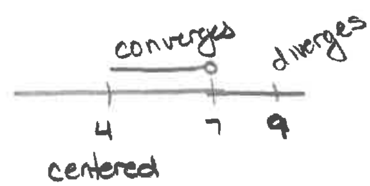
21. If the power series $\sum_{n=0}^{\infty} a_n(x-4)^n$ converges at $x = 7$ and diverges at $x = 9$, which of the following must be true?

~~I.~~ The series converges at $x = 1$.

II. The series converges at $x = 2$.

~~III.~~ The series diverges at $x = -1$.



$1 < x < 7$

converges

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(A) I only

(B) II only

(C) I and II only

(D) II and III only

$e^{-x} = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \frac{x^4}{4!} \ldots$

$\frac{1}{2}(e^x + e^{-x}) = \frac{1}{2}(2 + 2x^2/2! + 2x^4/4! \ldots)$

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22. A function f has Maclaurin series given by $1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \cdots + \frac{x^{2n}}{(2n)!} + \cdots$ .Which of the following is an expression forf(x) ?

~~(A)~~ $\cos x$

$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!}$

~~(B)~~ $e^x - \sin x$

~~(C)~~ $e^x + \sin x$

won't cancel out + terms

$e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!}$

$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!}$

$e^x - \sin x = 1 + \frac{x^2}{2} + \frac{2x^3}{3!}$

(D) $\frac{1}{2}(e^x + e^{-x})$

(E) e^{x^2}

$e^{x^2} = 1 + x^2 - \frac{x^6}{3!}$

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23. Which of the following is the Maclaurin series for the function f defined by $f(x) = 1 + x^2 + \cos x$?

$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!}$

$1 + x^2 + \cos x = 2 + \frac{x^2}{2} + \frac{x^4}{4!} - \frac{x^6}{6!}$

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(A) $2+\frac{x^2}{2}+\frac{x^4}{24}+\cdots$

(B) $2+\frac{3x^2}{2}+\frac{x^4}{24}+\cdots$

(C) $1+x+x^2-\frac{x^3}{6}+\cdots$

(D) $2+x+\frac{3x^2}{2}+\frac{x^3}{6}+\cdots$