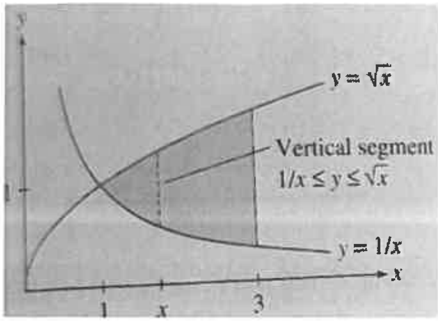
1. Evaluate $\iint_D x^2y\,dA$, where D is the region in the figure below:



$$\int_1^3\int_{1/x}^{\sqrt{x}} x^2y\,dy\,dx$$

$$=\int_1^3\left[\tfrac12 x^2y^2\Big|_{1/x}^{\sqrt{x}}\right]dx$$

$$=\int_1^3\left[\tfrac12 x^2(\sqrt{x})^2-\tfrac12 x^2\left(\tfrac1x\right)^2\right]dx$$

$1\le x\le 3$

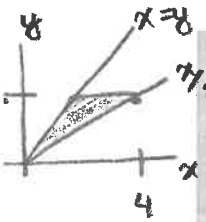
$\frac1x\le y\le\sqrt{x}$ ← inner integral

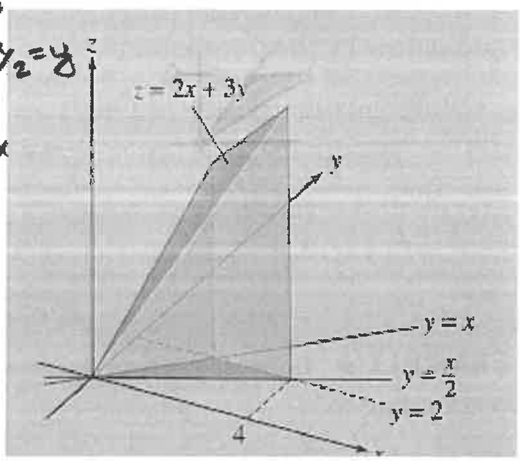
$$=\int_1^3 \tfrac12 x^3-\tfrac12\,dx=\tfrac18 x^4-\tfrac12 x\Big|_1^3$$

$$=\tfrac18(81)-\tfrac32-\tfrac18+\tfrac12$$

$\boxed{=9}$

2. Find the volume V of the region between the plane $z = 2x + 3y$ and the triangle D in the figure below:





$$\int_0^2\int_y^{2y}(2x+3y)\,dx\,dy$$

$$\int_0^2\left[x^2+3yx\Big|_y^{2y}\right]dy$$

$$\int_0^2\left[4y^2+6y^2-(y^2+3y^2)\right]dy$$

$$\int_0^2 6y^2\,dy$$

$0<y<2$ $\quad$ $0<x<4$

$y<x<2y$ $\quad$ OR * y would be split in two

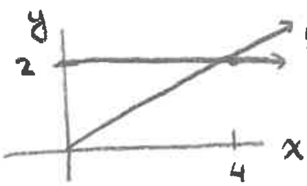
$$2y^3\Big|_0^2=2(8-0)=\boxed{16}$$

3. Evaluate $\iint_D e^{y^2}\,dA$ for D: $0\le x\le 4$, $\frac12 x\le y\le 2$

$$\int_0^4\int_{1/2x}^2 e^{y^2}\,dy\,dx$$

* doesn't work b/c can't $\int e^{y^2}$

so rewrite bounds:



$0<y<2$

$0<x<2y$

$$\int_0^2\int_0^{2y} e^{y^2}\,dx\,dy$$

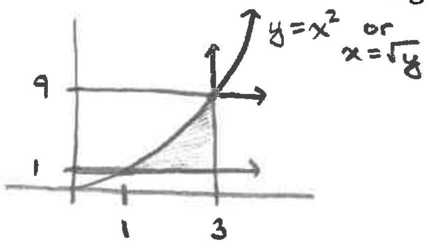
$$=\int_0^2 e^{y^2}x\Big|_0^{2y}\,dy$$

$$=\int_0^2\left[2ye^{y^2}-0\right]dy$$

$$=e^{y^2}\Big|_0^2$$

$$\boxed{e^4-1}$$

4. Sketch the domain of integration D corresponding to $\int_1^9 \int_{\sqrt{y}}^3 xe^y\,dxdy$ then change the order of integration and evaluate.



$1 < y < x^2$

$1 < x < 3$

$$\int_1^3 \int_1^{x^2} xe^y \, dy\, dx$$

$$\int_1^3 xe^y \Big|_1^{x^2} dx$$

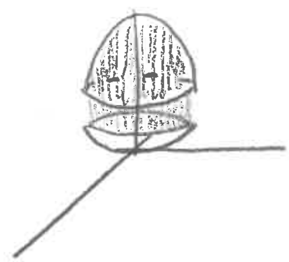
$$\int_1^3 (xe^{x^2} - xe)\, dx$$

$$\tfrac{1}{2}e^{x^2} - \tfrac{1}{2}e x^2 \Big|_1^3$$

$$\tfrac{1}{2}e^9 - \tfrac{9}{2}e - \tfrac{1}{2}e + \tfrac{1}{2}e$$

$$\boxed{\tfrac{1}{2}(e^9 - 9e)}$$

5. Find the volume V of the solid bounded above and below by the paraboloids corresponding to $z = 8 - x^2 - y^2$ and $z = x^2 + y^2$ and lying over the domain $D = \{(x, y):\ -1 \le x \le 1,\ -1 \le y \le 1\}$ * area above - below *



$$\int_{-1}^1 \int_{-1}^1 (8 - x^2 - y^2) - (x^2 + y^2)\, dx\, dy$$

$$= \int_{-1}^1 \int_{-1}^1 8 - 2x^2 - 2y^2\, dx\, dy$$

$$= \int_{-1}^1 8x - \tfrac{2}{3}x^3 - 2y^2 x \Big|_{-1}^1 dy$$

$$= \int_{-1}^1 \left[8 - \tfrac{2}{3} - 2y^2 - (-8 + \tfrac{2}{3} + 2y^2\right] dy$$

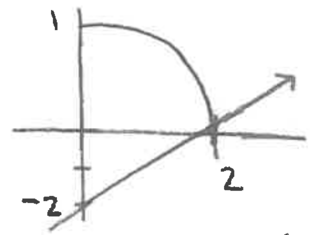
$$= \int_{-1}^1 16 - 4/3 - 4y^2\, dy$$

$$= 44/3\, y - 4/3\, y^3 \Big|_{-1}^1 = 44/3 - 4/3 - (-44/3 + 4/3)$$

$$= \boxed{80/3}$$

6. Evaluate the following:

a. $\int_0^1 \int_{x-2}^{\cos \pi x} y\,dy\,dx$



$\int_0^1 \frac{1}{2}y^2 \Big|_{x-2}^{\cos \pi x} dx$

can't change order of bounds to integrate

$\int_0^1 \frac{1}{2}(\cos \pi x)^2 - \frac{1}{2}(x-2)^2\, dx$

$\frac{1}{2}\int_0^1 (\cos \pi x)^2 dx - \frac{1}{6}(x-2)^3 \Big|_0^1$

$\frac{1}{2}\int_0^1 (\cos \pi x)^2 dx - \left(-\frac{1}{6} + \frac{8}{6}\right)$

$\frac{1}{2}\int_0^1 (\cos \pi x)^2 dx - \frac{7}{6}$

$= \frac{1}{2} \cdot \frac{1}{2} - \frac{7}{6}$

$= -\frac{25}{24}$

* could approximate w/ maclaurin series
calc will give answer

b. $\int_0^7 \int_0^{\sqrt{y}} 2x \cos x^2\, dx\,dx$

$\int_0^7 \sin x^2 \Big|_0^{\sqrt{y}} dy$

$\int_0^7 \sin y - \sin 0\, dy$

$-\cos y \Big|_0^7$

$-\cos 7 + \cos 0$

$\boxed{-\cos 7 + 1}$

c. $\int_0^8 \int_0^{-x+8} (2x + 5y)^2 dydx$

$$\int_0^8 \frac{1}{5}(2x+5y)^3 \frac{1}{3} \Big|_0^{-x+8} dx$$

$$= \frac{1}{15}\int_0^8 (2x + 5(-x+8))^3 - (2x)^3 \, dx$$

$$= \frac{1}{15}\int_0^8 \left[(-3x+40)^3 - 8x^3\right] dx$$

$$= \frac{1}{15}\left[-\frac{1}{3}(-3x+40)^4 \frac{1}{4} - 8x^4 \frac{1}{4} \Big|_0^8 \right]$$

$$= \frac{1}{15}\left[-\frac{1}{12}(-24+40)^4 - 2(8)^4 - \left(-\frac{1}{12}(40)^4 - 0\right)\right.$$

$$= \boxed{13,312}$$