

16.4 Integration in Cylindrical Coordinates
Multivariable Calculus

Triple Integrals in Cylindrical Coordinates

$$\int \int \int_D f(r, \theta, z) dV = \int_{\theta=\alpha}^{\theta=\beta} \int_{r=h_1(\theta)}^{r=h_2(\theta)} \int_{z=g_1(r,\theta)}^{z=g_2(r,\theta)} f(r, \theta, z) dz r dr d\theta.$$

$$x = r \cos \theta, \quad y = r \sin \theta, \quad z = z$$

$$r^2 = x^2 + y^2, \quad \tan \theta = y/x.$$

1. Evaluate $\int \int \int_E 4xy dV$ where E is the region bounded by $z = 2x^2 + 2y^2 - 7$ and $z = 1$

2. Evaluate $\int \int \int_E e^{-x^2-z^2} dV$ where E is the region between the two cylinders $x^2 + z^2 = 4$ and $x^2 + z^2 = 0$ with $1 \leq y \leq 5$ and $z \leq 0$

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3. Evaluate $\iiint_E zdV$ where E is the region bounded by $x + y + z = 2$ and $x = 0$ and inside the cylinder $y^2 + z^2 = 1$

4. Use a triple integral to determine the volume of the region below $z = 6 - x$, above $z = -\sqrt{4x^2 + 4y^2}$ inside the cylinder $x^2 + y^2 = 3$ with $x \leq 0$

5. Evaluate the following integral by first converting to an integral in cylindrical coordinates.

$$\int_0^{\sqrt{5}} \int_{-\sqrt{5-x^2}}^0 \int_{x^2+y^2-11}^{9-3x^2-3y^2} 2x - 3y \, dz \, dy \, dx$$