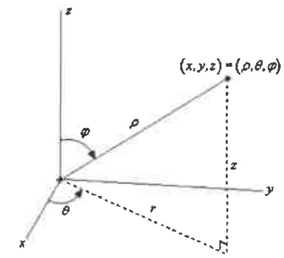


Conversion formulas for spherical coordinates:

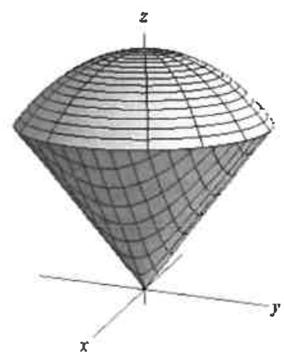
$$x = \rho \sin\varphi \cos\theta \qquad y = \rho \sin\varphi \sin\theta \qquad z = \rho\cos\varphi$$

$$x^2 + y^2 + z^2 = \rho^2$$

$$\rho \geq 0 \qquad 0 \leq \varphi \leq \pi$$

* phi can be φ or ϕ

We will be slicing our solid into spherical wedges to calculate the integrals:

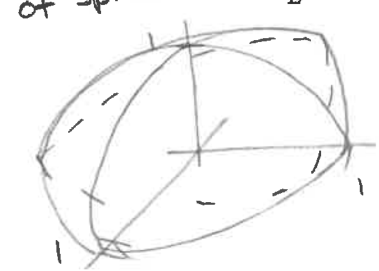


Triple Integrals in Spherical Coordinates:

$$\iiint_E f(x,y,z)\, dV = \int_\delta^\gamma \int_\alpha^\beta \int_a^b \rho^2 \sin\varphi \; f(\rho\sin\varphi\cos\theta, \rho\sin\varphi\sin\theta, \rho\cos\varphi)\, d\rho\, d\theta\, d\varphi$$

half of sphere

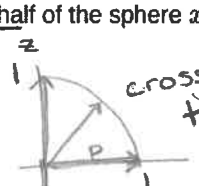
1) Evaluate $\iiint_E 16z\, dV$ where E is the upper half of the sphere $x^2 + y^2 + z^2 = 1$.



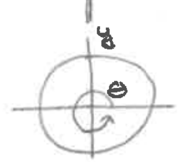
$0 \leq \rho \leq 1$

$0 \leq \theta \leq 2\pi$

$0 \leq \varphi \leq \pi/2$



cross section to see ρ



cross section to see θ

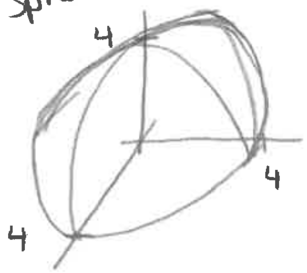
$16z = 16\rho\cos\varphi$

$$\int_0^{\pi/2} \int_0^{2\pi} \int_0^1 \rho^2 \sin\varphi \, 16\rho\cos\varphi \, d\rho\, d\theta\, d\varphi$$

$$\boxed{= 4\pi}$$

2) Evaluate $\iiint_E 10xz + 3\, dV$ where E is the region portion of $x^2 + y^2 + z^2 = 16$ with $z \geq 0$.

top half of sphere



$10xz + 3 = 10\rho\sin\varphi\cos\theta\,\rho\cos\varphi + 3$

$= 10\rho^2\cos\theta\sin\varphi\cos\varphi + 3$

$$\int_0^{\pi/2}\int_0^{2\pi}\int_0^4 \rho^2\sin\varphi\left(10\rho^2\cos\theta\sin\varphi\cos\varphi + 3\right)d\rho\, d\theta\, d\varphi$$

$0 \leq \rho \leq 4$

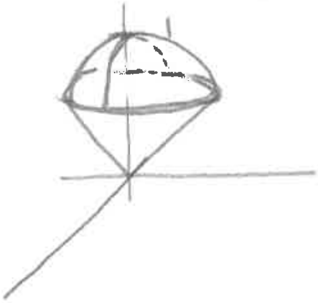
$0 \leq \varphi \leq \pi/2$

$0 \leq \theta \leq 2\pi$

$\boxed{= 128\pi}$

3) Evaluate $\iiint_E 3z\, dV$ where E is the region inside both $x^2 + y^2 + z^2 = 1$ and $z = \sqrt{x^2 + y^2}$.

★ answer on other page ★



$0 \leq \rho \leq 1$
$0 \leq \theta \leq 2\pi$ } correspond to radius of sphere & how much sphere when rotate around z

in cylindrical coordinates $x^2 + y^2 = r^2$ or $\sqrt{x^2+y^2} = r$

in spherical coordinates $r = \rho\sin\varphi$

$z = \sqrt{x^2+y^2}$

$\rho\cos\varphi = r = \rho\sin\varphi$

$\frac{\rho\cos\varphi}{\rho\sin\varphi} = 1$

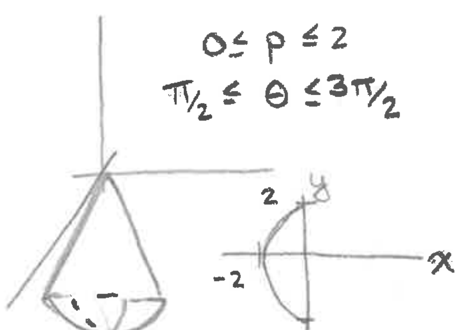
$\tan\varphi = 1 \quad \varphi = \pi/4 \quad 0 \leq \varphi \leq \pi/4$

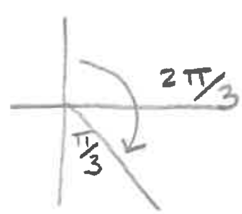
4) Evaluate $\iiint_E zx\, dV$ where E is inside both $x^2 + y^2 + z^2 = 4$ and the cone (pointing upward) that makes an angle of $\frac{\pi}{3}$ with the negative z-axis and has $x \leq 0$.

★ φ measures from positive z-axis

$0 \leq \rho \leq 2$

$\pi/2 \leq \theta \leq 3\pi/2$





$2\pi/3 \leq \varphi \leq \pi$

$$\int_{2\pi/3}^{\pi}\int_{\pi/2}^{3\pi/2}\int_0^2 \rho^2\sin\varphi\left(\rho\cos\varphi\,\rho\sin\varphi\cos\theta\right)d\rho\, d\theta\, d\varphi$$

$= \boxed{\frac{8\sqrt{3}}{5}}$

3 continued)

$$\int_0^{\pi/4}\int_0^{2\pi}\int_0^{1} \rho^2 \sin\varphi \left(3\rho\cos\varphi\right) d\rho\, d\theta\, d\varphi$$

$$= \boxed{\frac{3\pi}{8}}$$

proof $r = \rho\sin\varphi$

$$r = \sqrt{x^2+y^2}$$

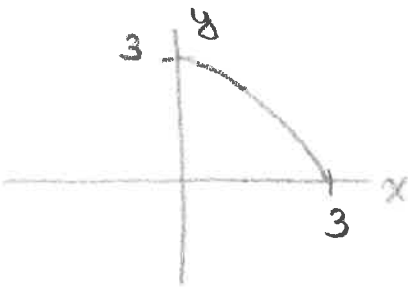
$$= \sqrt{\rho^2\sin^2\varphi\cos^2\theta + \rho^2\sin^2\varphi\sin^2\theta}$$

$$= \sqrt{\rho^2\sin^2\varphi\left(\cos^2\theta + \sin^2\theta\right)}$$

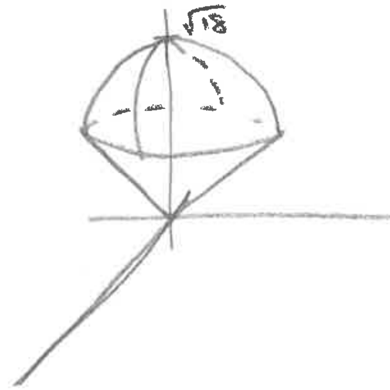
$$= \sqrt{\rho^2\sin^2\varphi}$$

$$= \rho\sin\varphi$$

Convert $\int_0^3 \int_0^{\sqrt{9-y^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{18-x^2-y^2}} x^2+y^2+z^2\, dz\, dx\, dy$ into spherical coordinates.



$z = \sqrt{x^2+y^2}$ $\qquad$ $z = \sqrt{18-x^2-y^2}$



$0 \le \rho \le \sqrt{18}$

$0 \le \theta \le \pi/2$

$z = \sqrt{x^2+y^2}$

$\rho\cos\varphi = r$

$= \rho\sin\varphi$

$\dfrac{\rho\cos\varphi}{\rho\sin\varphi} = 1$

$\tan\varphi = 1$

$\varphi = \pi/4$

$$\boxed{\int_0^{\pi/4} \int_0^{\pi/2} \int_0^{\sqrt{18}} \rho^2 \sin\varphi \left(\rho^2\right) d\rho\, d\theta\, d\varphi}$$