

[https://www.youtube.com/watch?v=WA5\\_a3C2iqY&list=PLHXZ9OQGMqxfW0GMqeUE1bLKaYor6kbHa&index=3&t=612s](https://www.youtube.com/watch?v=WA5_a3C2iqY&list=PLHXZ9OQGMqxfW0GMqeUE1bLKaYor6kbHa&index=3&t=612s)

$$\int_C f(x, y) ds = \int_a^b f(h(t), g(t)) \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt$$

Curve	Parametric Equations	
$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ (Ellipse)	Counter-Clockwise	Clockwise
	$x = a \cos(t)$ $y = b \sin(t)$ $0 \leq t \leq 2\pi$	$x = a \cos(t)$ $y = -b \sin(t)$ $0 \leq t \leq 2\pi$
$x^2 + y^2 = r^2$ (Circle)	Counter-Clockwise	Clockwise
	$x = r \cos(t)$ $y = r \sin(t)$ $0 \leq t \leq 2\pi$	$x = r \cos(t)$ $y = -r \sin(t)$ $0 \leq t \leq 2\pi$
$y = f(x)$	$x = t$ $y = f(t)$	
$x = g(y)$	$x = g(t)$ $y = t$	

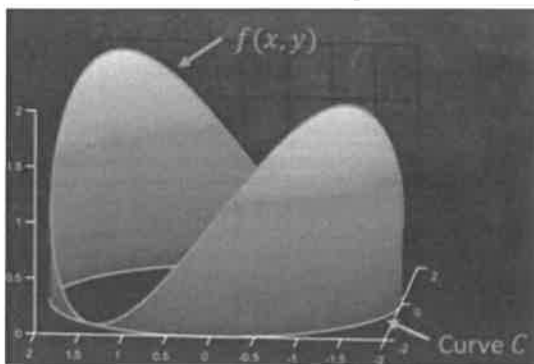
$$\vec{r}(t) = (1-t)\langle x_0, y_0, z_0 \rangle + t\langle x_1, y_1, z_1 \rangle, \quad 0 \leq t \leq 1$$

Line Segment From  
 $(x_0, y_0, z_0)$  to  
 $(x_1, y_1, z_1)$

or

$$\begin{aligned} x &= (1-t)x_0 + tx_1 \\ y &= (1-t)y_0 + ty_1 \\ z &= (1-t)z_0 + tz_1 \end{aligned}, \quad 0 \leq t \leq 1$$

1. Calculate the line integral of  $f(x, y) = \frac{x^2+y^2}{4} + \frac{xy}{2}$  above the circle of radius 2 centered at the origin.



$$x^2 + y^2 = 2^2$$

$$x = 2 \cos t \quad y = 2 \sin t$$

$$\vec{r}(t) = 2 \cos t \vec{i} + 2 \sin t \vec{j} \quad t \in [0, 2\pi]$$

$$\begin{aligned} f(2 \cos t, 2 \sin t) &= \frac{(2 \cos t)^2 + (2 \sin t)^2}{4} + \frac{(2 \cos t)(2 \sin t)}{2} \\ &= \frac{4}{4} + \frac{2 \sin 2t}{2} \quad \text{*double angle} \\ &= 1 + \sin 2t \end{aligned}$$

$\sin 2\theta = 2 \sin \theta \cos \theta$

$$\int_0^{2\pi} (1 + \sin 2t) \sqrt{(-2 \sin t)^2 + (2 \cos t)^2} dt$$

$$= 4\pi$$

2. Evaluate  $\int_C xy^4 ds$  where  $C$  is the right half of the circle  $x^2 + y^2 = 16$  traced out in a counter-clockwise direction.

$$x = 4 \cos t \quad y = 4 \sin t \quad t \in [-\pi/2, \pi/2]$$

$$\vec{r}(t) = 4 \cos t \vec{i} + 4 \sin t \vec{j}$$

$$\int_{-\pi/2}^{\pi/2} 1024 \cos t \sin^4 t \sqrt{(-4 \sin t)^2 + (4 \cos t)^2} dt$$

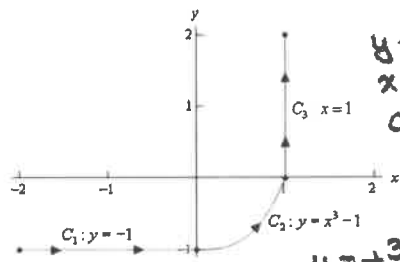
$$f(x, y) = xy^4$$

$$\begin{aligned} f(4 \cos t, 4 \sin t) &= 4 \cos t (4 \sin t)^4 \\ &= 1024 \cos t \sin^4 t \end{aligned}$$

$$\int_{-\pi/2}^{\pi/2} 1024 \cos t \sin^4 t (4) dt$$

$$= \boxed{\frac{8192}{5}} \approx 1638.4$$

3. Evaluate  $\int_C 4x^3 ds$  where  $C$  is the curve shown below.



$$y = t$$

$$x = 1$$

$$0 \leq t \leq 2$$

$$f(1, t) = 4$$

0 to 2 \*  
b/c t in terms  
of y not x

$$y = t^3 - 1$$

$$x = t$$

$$0 \leq t \leq 1$$

$$f(t, t^3 - 1) = 4t^3$$

$$y = -1$$

$$x = t$$

$$-2 \leq t \leq 0$$

$$f(t, -1) = 4t^3$$

Need to split into 3  
integrals

$$\int_C 4x^3 ds = \int_{-2}^0 4t^3 \sqrt{(1)^2 + (0)^2} dt + \int_0^1 4t^3 \sqrt{1^2 + (3t^2)^2} dt + \int_0^2 4\sqrt{0+1^2} dt$$

$$= -16 + 2.268 + 8$$

$$= \boxed{-5.732}$$

4. Evaluate  $\int_C 4x^3 ds$  where  $C$  is the line segment from  $(-2, -1)$  to  $(1, 2)$

$$\vec{r}(t) = (1-t)\langle -2, -1 \rangle + t\langle 1, 2 \rangle \quad 0 \leq t \leq 1$$

$$= \langle -2 + 3t, -1 + 3t \rangle$$

$$x = -2 + 3t \quad y = -1 + 3t$$

$$f(-2+3t, -1+3t) = 4(-2+3t)^3$$

$$\int_C 4x^3 ds = \int_0^1 4(-2+3t)^3 \sqrt{(3)^2 + (3)^2} dt$$

$$= \boxed{-21.213}$$

5. Evaluate  $\int_C 4x^3 ds$  where  $C$  is the line segment from  $(1, 2)$  to  $(-2, -1)$

$$\vec{r}(t) = (1-t)\langle 1, 2 \rangle + t\langle -2, -1 \rangle$$

$$= \langle 1-3t, 2-3t \rangle \quad 0 \leq t \leq 1$$

$$x = 1-3t \quad y = 2-3t$$

$$f(1-3t, 2-3t) = 4(1-3t)^3$$

$$\int_0^1 4(1-3t)^3 \sqrt{(-3)^2 + (-3)^2} dt$$

$$= \boxed{-21.213}$$

