Sequence: is a list of numbers written in an explicit order.

Primary focus is on infinite sequences and whether or not they converge or diverge. If a sequences converges its terms approach limiting values.

Limit of a Sequence

We write $\lim_{n \to \infty} a_n = L$ and say that the sequence converges to L. Sequences that do not have limits diverge

Properties of Limits

If L and M are real numbers and $\lim_{n \to \infty} a_n = L$ and $\lim_{n \to \infty} b_n = M$, then:

1. Sum Rule:
   $\lim_{n \to \infty}(a_n + b_n) = L + M$
2. Product Rule:
   $\lim_{n \to \infty}(a_n b_n) = L \cdot M$
3. Quotient Rule:
   $\lim_{n \to \infty} \frac{a_n}{b_n} = \frac{L}{M}$
4. Difference Rule:
   $\lim_{n \to \infty}(a_n - b_n) = L - M$
5. Constant Multiple Rule:
   $\lim_{n \to \infty}(c \cdot a_n) = c \cdot L$

1. Determine whether the sequence converges or diverges. If it converges, find its limit.

a. $a_n = \frac{2n-1}{n}$

$$\lim_{n\to\infty} \frac{2n-1}{n} = \lim_{n\to\infty}\left(2-\frac{1}{n}\right)$$

$$= \lim_{n\to\infty} 2 - \lim_{n\to\infty}\frac{1}{n}$$

$$= 2 - 0$$

$$= 2$$

$\boxed{\text{sequence converges to } 2}$

b. $a_n = \frac{n}{n^2+1}$

$$\lim_{n\to\infty}\frac{n}{n^2+1}$$

$$= 0$$

$\boxed{\text{sequence converges to } 0}$

c. $a_n = (-1)^n \frac{n+1}{n^2+2}$

$$\lim_{n\to\infty} (-1)^n \frac{n+1}{n^2+2}$$

$$= \lim_{n\to\infty}(-1)^n \lim_{n\to\infty}\frac{n+1}{n^2+2}$$

$$= (\pm 1)(0)$$

$$= 0$$

$\boxed{\text{sequence converges to } 0}$

d. $a_n = (0.9)^n$

$$\lim_{n\to\infty}(0.9)^n = \lim_{n\to\infty}\left(\frac{9}{10}\right)^n$$

* fraction will get smaller & smaller as $n\to\infty$

$$= 0$$

$\boxed{\text{sequence converges to } 0}$

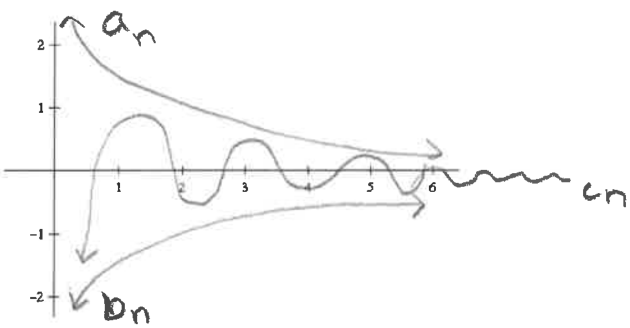
e. $a_n = \cos(n\frac{\pi}{2})$

$$\lim_{n\to\infty}\cos\left(\frac{\pi}{2}n\right)$$

$\boxed{\text{diverges}}$

Squeeze Theorem for Sequences

If $\lim_{n\to\infty} a_n = \lim_{n\to\infty} c_n = L$ and if there is an integer N for which $a_n \le b_n \le c_n$ for all $n > N$, then $\lim_{n\to\infty} b_n = L$



c_n is trapped in between a_n and b_n since both limits approach 0 so does c_n

2. Use the Squeeze Theorem to show that the sequence with given nth term converges and find its limit.

a. $a_n = \frac{1}{2^n}$

$$\frac{1}{2^n} < \frac{1}{n}$$

$$-\frac{1}{n} \le \frac{1}{2^n} \le \frac{1}{n}$$

$$\lim_{n\to\infty} -\frac{1}{n} \le \lim_{n\to\infty} \frac{1}{2^n} \le \lim_{n\to\infty} \frac{1}{n}$$

$$0 \le \lim_{n\to\infty} \frac{1}{2^n} \le 0$$

$\boxed{\text{converges to } 0}$

b. $a_n = \frac{\sin^2 n}{2^n}$

$$|\sin^2 n| \le 1$$

$$\left|\frac{\sin^2 n}{2^n}\right| \le \frac{|\sin^2 n|}{|2^n|} \le \frac{1}{2^n}$$

$$-\frac{1}{n} \le \frac{\sin^2 n}{2^n} \le \frac{1}{2^n}$$

$$\lim_{n\to\infty} -\frac{1}{n} \le \lim_{n\to\infty} \frac{\sin^2 n}{2^n} \le \lim_{n\to\infty} \frac{1}{2^n}$$

$$0 \le \lim_{n\to\infty} \frac{\sin^2 n}{2^n} \le 0$$

$\boxed{\text{converges to } 0}$

Absolute Value Theorem

Consider the sequence $\{a_n\}$. If $\lim_{n\to\infty} |a_n| = 0$, then $\lim_{n\to\infty} a_n = 0$