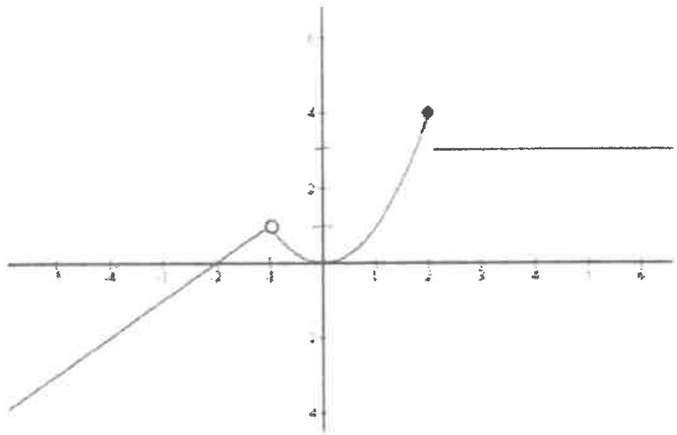
**No Calculator**

1. Use the graph below to help answer the following questions:



a. $f(-2) =$

b. $\lim_{x \to -2} f(x) =$

c. $f(-1) =$

d. $\lim_{x \to -1^-} f(x) =$

e. $\lim_{x \to -1^+} f(x) =$

f. $\lim_{x \to -1} f(x) =$

g. $f(2) =$

h. $\lim_{x \to 2^-} f(x) =$

i. $\lim_{x \to 2^+} f(x) =$

j. $\lim_{x \to 2} f(x) =$

k. Name the type of discontinuities and where they occur.

41. If $\lim_{x \to a} f(x) = L$, where $L$ is a real number, which of the following must be true? 1985 AB 41

(A) $f'(a)$ exists.

(B) $f(x)$ is continuous at $x = a$.

(C) $f(x)$ is defined at $x = a$.

(D) $f(a) = L$

(E) None of the above

4. Assume $\lim_{x\to b} f(x) = 7$ and $\lim_{x\to b} g(x) =-3$

a. $\lim_{x\to b}(f(x)+g(x)) =$

c. $\lim_{x\to b}(f(x)*g(x)) =$

b. $\lim_{x\to b} 4g(x) =$

d. $\lim_{x\to b}\left(\frac{f(x)}{g(x)}\right) =$

5. Evaluate the following limits:

a. $\lim_{x\to 7}\sec\left(\frac{\pi x}{6}\right) =$

d. $\lim_{y\to 2}\frac{y^2+5y+6}{y+2} =$

b. $\lim_{x\to 4}\sqrt[3]{x+4} =$

e. $\lim_{x\to -2}(x-6)^{\frac{2}{3}} =$

f. $\lim_{x\to 2}\sqrt{x+3} =$

c. $\lim_{x\to \frac{1}{2}} 3x^2(2x-1) =$

6. Evaluate the following limits

a. $\lim_{x\to 2^+} f(x)$, if $f(x) = \begin{cases} 3x+1, & x<2 \\ \frac{5}{x+1}, & x\geq 2 \end{cases}$

b. $\lim\limits_{x \to 1} \frac{x^2-4}{x-1}$

c. $\lim\limits_{x \to 2} \frac{x+1}{x^2-4}$

7. Evaluate, show your work:

a. $\lim\limits_{x \to 3} \frac{x^2-x-6}{x-3}$

d. $\lim\limits_{x \to 4} \frac{\sqrt{x+5}-3}{x-4}$

b. $\lim\limits_{x \to 0} \frac{\frac{1}{2+x}-\frac{1}{2}}{x}$

e. $\lim\limits_{x \to 1} \frac{x-1}{x^2-1}$

c. $\lim\limits_{x \to 0} \frac{\sqrt{2x+1}-1}{x}$

f. $\lim\limits_{x \to 0} \frac{(4+x)^2-16}{x}$

g. $\lim_{t \to 2} \frac{t^2-3t+2}{t^2-4}$

h. $\lim_{x \to 0} \frac{(2+x)^3-8}{x}$

8. Evaluate each of the following limits analytically. Be sure to show all steps in your evaluation.

a. $\lim_{x \to 0} \frac{\sin x}{5x}$

b. $\lim_{x \to 0} \frac{\sin 5x}{x}$

9. Evaluate each of the following by combining properties of limits and your algebra skills.

a. $\lim_{x \to 0} \frac{x+\sin x}{x}$

c. $\lim_{x \to 0} \frac{\sin x}{2x^2-x}$

d. $\lim_{x \to 0} \frac{\sin^2 x}{x}$

b. $\lim_{x \to 0} \frac{\tan x}{x}$

10. Evaluate each limit

a. $\lim\limits_{x \to 0} \dfrac{\frac{3}{4-x} - \frac{3}{4}}{x}$

b. $\lim\limits_{x \to 0} \dfrac{5x^3+8x^2}{3x^4-16x^2}$

c. $\lim\limits_{x \to 3} \dfrac{\sqrt{x+1}}{x-4}$

d. $\lim\limits_{x \to 0} \dfrac{x^2-3x}{x}$

e. $\lim\limits_{x \to 1} \dfrac{x}{x^2-x}$

f. $\lim\limits_{x \to 0} \dfrac{\sin 2x}{x}$

g. $\lim\limits_{x \to 0} \dfrac{\sin 7x}{3x}$

h. $\lim\limits_{x \to 4} \dfrac{x^2-5x+4}{x^2-2x-8}$

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1998 AB 12

If $f(x) = \begin{cases} \ln x & \text{for } 0 < x \le 2 \\ x^2 \ln 2 & \text{for } 2 < x \le 4, \end{cases}$ then $\lim\limits_{x \to 2} f(x)$ is

(A) $\ln 2$ (B) $\ln 8$ (C) $\ln 16$ (D) 4 (E) nonexistent

11. Suppose $g(x) = \begin{cases} 2 - x, & x \leq 1 \\ \frac{x}{2} + 1, & x > 1 \end{cases}$

a. $\lim_{x \to 1^-} g(x) =$

c. $\lim_{x \to 1} g(x) =$

b. $\lim_{x \to 1^+} g(x) =$

d. $g(1) =$

13. For each of the following find:

a. $\lim_{x \to \infty} f(x)$

b. $\lim_{x \to -\infty} f(x)$

c. Identify all horizontal asymptotes, if any

i. $f(x) = \frac{x-2}{2x^2+3x-5}$

iii. $f(x) = \frac{3x^2-x+5}{x^2-4}$

v. $f(x) = \frac{|x|}{x}$

ii. $f(x) = \frac{4x^3-2x+1}{x^2-2x+1}$

iv. $f(x) = \frac{e^{-x}}{x}$

vi. $f(x) = \frac{\sin x}{2x^2+x}$

14. For each of the following:
    a. Find the vertical asymptotes of the graph of $f(x)$
    b. Describe the behavior of the graph of $f(x)$ to the left and right of each asymptote

i. $f(x) = \frac{1}{x-3}$ ii. $f(x) = \frac{1}{x^2-4}$ iii. $f(x) = \frac{1-x}{2x^2-5x-3}$

15. Find the limit of $g(x)$ as
    a. $x \to \infty$
    b. $x \to -\infty$
    c. $x \to 0^-$
    d. $x \to 0^+$

i. $g(x) = \begin{cases} \frac{1}{x}, & x < 0 \\ \frac{2x-3}{x+1}, & x \geq 0 \end{cases}$ ii. $g(x) = \begin{cases} \frac{3x}{x-1}, & x \leq 0 \\ \frac{1}{x^2}, & x > 0 \end{cases}$

16. Sketch a function that satisfies the stated conditions:

$\lim_{x \to 1} f(x) = 2$

$\lim_{x \to 5^-} f(x) = \infty$

$\lim_{x \to 5^+} f(x) = \infty$

$\lim_{x \to \infty} f(x) = -1$

$\lim_{x \to -\infty} f(x) = 0$

$\lim_{x \to -2^-} f(x) = \infty$

$\lim_{x \to -2^+} f(x) = -\infty$

17. Sketch a function that satisfies the stated conditions. Include any asymptotes.

$\lim_{x \to 2} f(x) = -1$

$\lim_{x \to 4^+} f(x) = -\infty$

$\lim_{x \to 4^-} f(x) = \infty$

$\lim_{x \to \infty} f(x) = \infty$

$\lim_{x \to -\infty} f(x) = 2$

18. Evaluate each limit

a. $\lim\limits_{x \to -\infty} \frac{x}{|x|}$

c. $\lim\limits_{x \to \infty} \frac{2x^2+1}{(2-x)(2+x)}$

e. $\lim\limits_{x \to -\infty} \frac{2^{-x}}{2^x}$

b. $\lim\limits_{x \to \infty} \frac{x}{|x|}$

d. $\lim\limits_{x \to \infty} \frac{2^{-x}}{2^x}$

f. $\lim\limits_{x \to -\infty} \frac{4-x^2}{x^2-1}$

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19. What are all horizontal asymptotes of the graph of $y = \dfrac{5+2^x}{1-2^x}$ in the $xy$-plane? 2008 AB 19

(A) $y = -1$ only

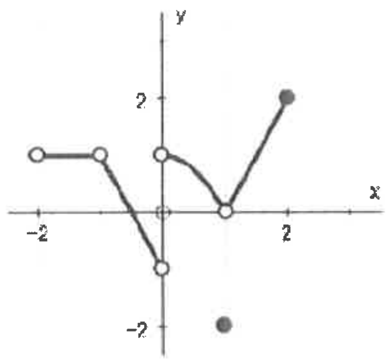
(B) $y = 0$ only

(C) $y = 5$ only

(D) $y = -1$ and $y = 0$

(E) $y = -1$ and $y = 5$

20. Use the function $g(x)$ defined and graphed below to answer the following questions:




$$g(x)=\begin{cases}1 & \text{if } -2<x<-1\\ -2x-1 & \text{if } -1<x<0\\ 1-x^2 & \text{if } 0<x<1\\ -2 & \text{if } x=1\\ 2x-2 & \text{if } 1<x\leq 2\end{cases}$$

a. Does $g(1)$ exist?

b. Does $\lim_{x\to 1} g(x)$ exist?

c. Does $\lim_{x\to 1} g(x) = g(1)$?

d. Is $g$ continuous at $x = 1$?

e. Is $g$ defined at $x =- 1$?

f. Is $g$ continuous at $x =- 1$?

g. For what values of $g$ continuous?

h. What value should be assigned to $g(-1)$ to make the extended function continuous at $x =- 1$?

i. What new value should be assigned to $g(1)$ to make the new function continuous at $x = 1$?

j. Is it possible to extend $g$ to be continuous at $x = 0$? If so, what value should the extended function have there? If not, why not?

21. Let $f(x) = \begin{cases} x^2 - 1, & x < 3 \\ 2ax, & x \geq 3 \end{cases}$ Find a value of $a$ so that the function $f$ is continuous. **Using the definition of continuity**. Justify your response.

22. Let $f(x) = \begin{cases} x^2 - a^2x, & x < 2 \\ 4 - 2x^2, & x \geq 2 \end{cases}$ Find a value of $a$ so that the function $f$ is continuous. **Using the definition of continuity**. Justify your response.

23. Let $f$ be the function defined by the following:

$$f(x) = \begin{cases} \sin x, & x < 0 \\ x^2, & 0 \leq x < 1 \\ 2 - x, & 1 \leq x < 2 \\ x - 3, & x \geq 2 \end{cases}$$

For what values of $x$ is $f$ NOT continuous? Use the definition of continuity to explain why.

24. Without using a picture, give a **written** explanation of why the function $f(x) = x^2 - 4x + 3$ has a zero in the interval $[2,\ 4]$.

25. Without using a picture, give a **written** explanation of why the function $f(x) = x^2 + 2x - 3$ must equal 3 at least once in the interval $[0,\ 2]$.

26. Let $g(x) = \frac{x^2+5x+6}{x^2+7x+10}$

a. Find the domain of $g(x)$

b. Find the $\lim_{x \to c} g(x)$ for all values of $c$ where $g(x)$ is not defined.

c. Find any horizontal asymptotes and justify your response.

d. Find any vertical asymptotes and justify your response.

e. Write an extension to the function so that $g(x)$ is continuous at $x = -2$. **Use the definition of continuity** to justify your response.

27. Let $h(x) = \begin{cases} 3x^2 - 4, & x \leq 2 \\ 5 + 4x, & x > 2 \end{cases}$

a. What is $h(0)$?

b. What is $h(4)$?

c. On the interval $[0, 4]$ there is no value of $x$ such that $h(x) = 10$ even though $h(0) < 10$ and $h(4) > 10$. Explain why this result does not contradict the IVT.

28. At what points is the tangent to $f(x) = x^2 + 4x - 1$ horizontal?

29. Find the average rate of change of $f(x) = 1 + \sin x$ over the interval $[0, \frac{\pi}{2}]$

30. Let $f(x) = 2x - x^2$

Find

a. $f(3)$

b. $f(3 + h)$

c. $\frac{f(3+h)-f(3)}{h}$

d. Find the instantaneous rate of change of $f$ at $x = 3$.

31. Let $f(x) = x^2 - 3x$, and point $P = (1, f(1))$. Find:

a. The slope of the curve at point P.

b. The equation of the tangent line at the point P.

c. An equation of the normal at point P.

32. An object is dropped from the top of a 130m tower. It's height above ground after $t$ seconds is $130 - 4.9t^2$ m. How fast is the object falling 2 seconds after it is dropped?

| $x$ | 0 | 1 | 2 |
|---|---|---|---|
| $f(x)$ | 1 | $k$ | 2 |

1998 AB 26

The function $f$ is continuous on the closed interval $[0,2]$ and has values that are given in the table above. The equation $f(x)=\frac{1}{2}$ must have at least two solutions in the interval $[0,2]$ if $k=$

(A) 0 (B) $\frac{1}{2}$ (C) 1 (D) 2 (E) 3

$\lim_{x\to\infty}\frac{(2x-1)(3-x)}{(x-1)(x+3)}$ is

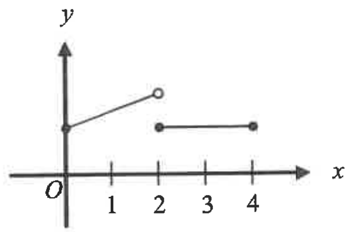
2008 AB 1

(A) $-3$ (B) $-2$ (C) 2 (D) 3 (E) nonexistent

$\lim_{n\to\infty}\frac{4n^2}{n^2+10{,}000n}$ is

1985 AB 5

(A) 0 (B) $\frac{1}{2{,}500}$ (C) 1 (D) 4 (E) nonexistent



Graph of $f$

2008 AB 77 Calc

The figure above shows the graph of a function $f$ with domain $0 \le x \le 4$. Which of the following statements are true?

I. $\lim_{x \to 2^-} f(x)$ exists.

II. $\lim_{x \to 2^+} f(x)$ exists.

III. $\lim_{x \to 2} f(x)$ exists.

(A) I only (B) II only (C) I and II only (D) I and III only (E) I, II, and III

$\lim_{x \to 0} \dfrac{5x^4 + 8x^2}{3x^4 - 16x^2}$ is

2008 AB 5

(A) $-\dfrac{1}{2}$ (B) 0 (C) 1 (D) $\dfrac{5}{3}+1$ (E) nonexistent

1969 AB 3

If $\begin{cases} f(x) = \dfrac{\sqrt{2x+5} - \sqrt{x+7}}{x-2}, & \text{for } x \neq 2, \\ f(2) = k \end{cases}$ and if $f$ is continuous at $x = 2$, then $k =$

(A) $0$ (B) $\frac{1}{6}$ (C) $\frac{1}{3}$ (D) $1$ (E) $\frac{7}{5}$

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1998 AB 83
Calc

If $a \neq 0$, then $\lim\limits_{x \to a} \dfrac{x^2 - a^2}{x^4 - a^4}$ is

(A) $\frac{1}{a^2}$ (B) $\frac{1}{2a^2}$ (C) $\frac{1}{6a^2}$ (D) $0$ (E) nonexistent