THEOREM 4 The Fundamental Theorem of Calculus, Part 1

If f is continuous on $[a, b]$, then the function

$$F(x) = \int_a^x f(t)\, dt$$

has a derivative at every point x in $[a, b]$, and

$$\frac{dF}{dx} = \frac{d}{dx}\int_a^x f(t)\, dt = f(x).$$

THEOREM 4 (continued) The Fundamental Theorem of Calculus, Part 2

If f is continuous at every point of $[a, b]$, and if F is any antiderivative of f on $[a, b]$, then

$$\int_a^b f(x)\, dx = F(b) - F(a).$$

This part of the Fundamental Theorem is also called the **Integral Evaluation Theorem.**

$$\frac{d}{dx}\int_a^x f(t)\, dt = f(x). \tag{1}$$

$$\int_a^b f(x)\, dx = F(b) - F(a).$$

1. If $g(x) = \int_{-2}^{x} w(t)dt$, then $g'(x) = ?$

$g'(x) = w(x)$

2. $\frac{d}{dx}\left[\int_{3}^{x}(5t^2 - 6t + 1)dt\right]$

$= 5x^2 - 6x + 1$

3. Let $g(x) = \int_{0}^{3x^2} \sin(t)\,dt$. What is $g'(x) = ?$

$= \sin(3x^2)(6x)$

$= 6x \sin(3x^2)$

4. Let $g(x) = \int_{1}^{\sin x} \sqrt{1+t^3}\,dt$. What is $g'(x) = ?$

$g'(x) = \sqrt{1+\sin^3 x}\ \cos x$

$= \cos x \sqrt{1+\sin^3 x}$

5. Find $\frac{d}{dx}\left[\int_{x^2}^{3x} f(t)dt\right]$

$= \frac{d}{dx}\left[\int_0^{3x} f(t)dt - \int_0^{x^2} f(t)dt\right]$

$= 3f(3x) - 2x\,f(x^2)$

6. Let $g(x) = \int_{5x}^{3x^2} \sqrt{1+t^3}\,dt$. What is $g'(x) = ?$

$g'(x) = \frac{d}{dx}\left[\int_0^{3x^2} \sqrt{1+t^3}\,dt - \int_0^{5x} \sqrt{1+t^3}\,dt\right]$

$= \sqrt{1+(3x^2)^3}\,(6x) - \sqrt{1+(5x)^3}\,(5)$

$= 6x\sqrt{1+27x^6} - 5\sqrt{1+125x^3}$

7. Evaluate

a. $\int_0^3 x^2 dx$

$= \frac{1}{3}x^3 \Big|_0^3$

$= \frac{1}{3}3^3 - \frac{1}{3}(0)^3$

$= \boxed{9}$

b. $\int_{\frac{\pi}{2}}^{\pi}(1+\cos x)dx$

$= x + \sin x \Big|_{\pi/2}^{\pi}$

$= (\pi + \sin\pi) - (\pi/2 + \sin \pi/2)$

$= \pi - \pi/2 - 1$

$= \boxed{\frac{\pi}{2} - 1}$

c. $\int_{-1}^{2} 3^x dx$

$= \frac{3^x}{\ln 3}\Big|_{-1}^{2}$

$= \frac{3^2}{\ln 3} - \frac{3^{-1}}{\ln 3}$

$= \frac{9}{\ln 3} - \frac{1}{3\ln 3} = \frac{27}{3\ln 3} - \frac{1}{3\ln 3}$

$= \frac{26}{3\ln 3}$

$= \boxed{\frac{26}{\ln 27}}$

d. $\int_4^9 f'(x)dx$

$= f(x)\Big|_4^9$

$= \boxed{f(9) - f(4)}$

8. Given $\frac{dy}{dx} = 3x^2 + 4x - 5$ with the initial condition $y(2) = -1$, find $y(3)$

☆ $\int_a^b f(x)dx = F(b) - F(a)$ ☆

$F(b) = F(a) + \int_a^b f(x)dx$

$y(3) = y(2) + \int_2^3 (3x^2+4x-5)dx$

$y(3) = -1 + (x^3 + 2x^2 - 5x)\Big|_2^3$

$-1 + (x^3 + 2x^2 - 5x)\Big|_2^3 = y(3)$

$-1 + \left[(3^3 + 2(3)^2 - 5(3)) - (2^3 + 2(2)^2 - 5(2))\right] = y(3)$

$-1 + (27 + 18 - 15 - 8 - 8 + 10) = y(3)$

$-1 + (24) = y(3)$

$\boxed{23 = y(3)}$

9. $f'(x) = \sin(x^2)$ and $f(2) = -5$. Find $f(1)$.

$$\int_1^2 f'(x)\,dx = f(2) - f(1)$$

$$f(1) = f(2) - \int_1^2 f'(x)\,dx$$

$$f(1) = -5 - \int_1^2 \sin(x^2)\,dx$$

$$\boxed{f(1) \approx -5.495}$$

Number 10

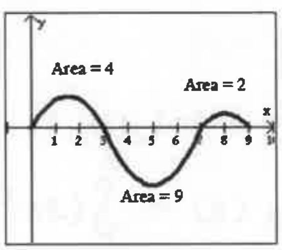
$$\int_3^9 f'(x)\,dx = f(9) - f(3)$$

$$f(9) = f(3) + \int_3^9 f'(x)\,dx$$

$$f(9) = 5 - 7$$

$$\boxed{f(9) = -2}$$

10. The graph of f' is shown at right, with areas of regions enclosed by the graph and the x-axis as indicated. Give that $f(3) = 5$, find $f(0)$, $f(7)$, and $f(9)$.



Graph of f'

$$\int_0^3 f'(x) = f(3) - f(0)$$

$$f(0) = f(3) - \int_0^3 f'(x)\,dx$$

$$f(0) = 5 - 4$$

$$\boxed{f(0) = 1}$$

$$\int_3^7 f'(x)\,dx = f(7) - f(3)$$

$$f(7) = f(3) + \int_3^7 f'(x)\,dx$$

$$f(7) = 5 - 9$$

$$\boxed{f(7) = -4}$$

11. A pizza with a temperature of $95°C$ is put into a $25°C$ room when $t = 0$. The pizza's temperature is decreasing at a rate of $r(t) = 6e^{-0.1t}\,°C$ per minute. Estimate the pizza's temperature when $t = 5$ minutes.

$R(0) = 95 \quad R(5) = ?$

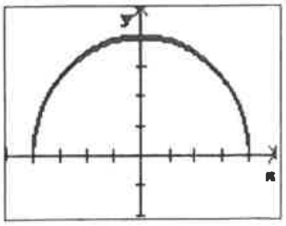
$$\int_0^5 r(t)\,dt = R(5) - R(0)$$

$$R(0) + \int_0^5 r(t)\,dt = R(5)$$

$$95 + \int_0^5 -6e^{-0.1t}\,dt = R(5)$$

$$\boxed{71.392°\ C = R(5)}$$

12. The graph of f' is the semicircle shown at the right. Find $f(-4)$ if $f(4) = 7$



$$\int_{-4}^{4} f'(x)dx = f(4) - f(-4)$$

$$f(-4) = f(4) - \int_{-4}^{4} f'(x)\,dx$$

$$f(-4) = 7 - \frac{1}{2}\pi(4)^2$$

$$\boxed{f(-4) = 7 - 8\pi}$$

The Trapezoidal Rule

To approximate $\int_a^b f(x)\,dx$, use

$$T = \frac{h}{2}\left(y_0 + 2y_1 + 2y_2 + \cdots + 2y_{n-1} + y_n\right),$$

where $[a, b]$ is partitioned into n subintervals of equal length $h = (b - a)/n$.
Equivalently,

$$T = \frac{\text{LRAM}_n + \text{RRAM}_n}{2},$$

where LRAM_n and RRAM_n are the Riemann sums using the left and right endpoints, respectively, for f for the partition.

Simpson's Rule

To approximate $\int_a^b f(x)\,dx$, use

$$S = \frac{h}{3}\left(y_0 + 4y_1 + 2y_2 + 4y_3 + \cdots + 2y_{n-2} + 4y_{n-1} + y_n\right),$$

where $[a, b]$ is partitioned into an *even* number n of subintervals of equal length $h = (b - a)/n$.

$f(0)=1$ $f(3/2)=23/8$

$f(1/2)=5/8$ $f(2)=7$

$f(1)=1$

5. Use the trapezoidal rule with $n = 4$ to approximate the value of $\int_0^2 (x^3 - x + 1)\,dx$.

height $= 1/2$

$= \frac{1}{2}\left(\frac{1}{2}\right)(f(0)+f(1/2)) + \frac{1}{2}\left(\frac{1}{2}\right)(f(1/2)+f(1)) + \frac{1}{2}\left(\frac{1}{2}\right)(f(1)+f(3/2)) + \frac{1}{2}\left(\frac{1}{2}\right)(f(3/2)+f(2))$

$= \frac{1}{4}\left(1 + 5/8 + 5/8 + 1 + 1 + 23/8 + 23/8 + 7\right)$

$= \boxed{4.25}$

6. The table below shows the velocity of a remote control car as it travelled down the hallway for 10 seconds. Using the trapezoidal rule, estimate the distance travelled by the car using 10 subintervals.

height $= 1$

Time (sec)	0	1	2	3	4	5	6	7	8	9	10
Velocity (in./sec)	0	6	10	16	14	12	18	22	12	4	2

$= \frac{1}{2}(1)(0+6) + \frac{1}{2}(1)(6+10) + \frac{1}{2}(1)(10+16) + \frac{1}{2}(1)(16+14) + \frac{1}{2}(1)(14+12) + \frac{1}{2}(1)(12+18) + \frac{1}{2}(1)(18+22) + \frac{1}{2}(1)(22+12) + \frac{1}{2}(1)(12+4) + \frac{1}{2}(1)(4+2)$

$= \boxed{116 \text{ in}}$

7. Use Simpson's Rule with $n = 4$ to approximate the value of $\int_1^2 \sin x\,dx$ and find the exact value of the integral to check your answer.

$= \frac{1/4}{3}\left(\sin(1) + 4\sin(5/4) + 2\sin(6/4) + 4\sin(7/4) + \sin(2)\right)$

$= \boxed{0.956}$

Most Difficult 1st

pg 310 #18, 67 pg. 320 #8