1. Write out the first four terms of the Maclaurin series of $f(x)$ if

$$f(0) = 2,\ f'(0) = 3,\ f''(0) = 4,\ f^{(3)}(0) = 12$$

$$P_3(x) = 2 + \frac{3(x-0)^1}{1!} + \frac{4(x-0)^2}{2!} + \frac{12(x-0)^3}{3!}$$

$$P_3(x) = 2 + 3x - 2x^2 + 2x^3$$

2. Find the terms through degree 4 of the Maclaurin series of $f(x) = \frac{\sin x}{1-x}$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots \qquad \frac{1}{1-x} = 1 + x + x^2 + x^3 + \dots$$

*geo w/ $a_1 = 1$, $r = x$

$$\frac{\sin x}{1-x} = \left(x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots\right)\left(1 + x + x^2 + x^3 + \dots\right)$$

$$P_4(x) = x + x^2 + x^3 + x^4 - \frac{x^3}{3!} - \frac{x^4}{3!} + \dots$$

$$P_4(x) = x + x^2 + \frac{5x^3}{6} + \frac{5x^4}{6}$$

3. Find the Taylor Series centered at $c = -1$ for $f(x) = e^{3x}$

$f(x) = e^{3x}$ $\quad f'(x) = 3e^{3x}$ $\quad f''(x) = 9e^{3x}$ $\quad f'''(x) = 27e^{3x}$

$f(-1) = e^{-3}$ $\quad f'(-1) = 3e^{-3}$ $\quad f''(-1) = 9e^{-3}$ $\quad f'''(-1) = 27e^{-3}$

$$T(x) = e^{-3} + 3e^{-3}(x+1) + \frac{9e^{-3}}{2!}(x+1)^2 + \frac{27e^{-3}}{3!}(x+1)^3$$

4. Show that for $|x| < 1$ $\tanh^{-1}x = x + \frac{x^3}{3} + \frac{x^5}{5} + \dots$

Hint: $\frac{d}{dx}\tanh^{-1}x = \frac{1}{1-x^2}$

$$\frac{d}{dx}\tanh^{-1}x = \frac{1}{1-x^2}$$

$$\int \frac{d}{dx}\tanh^{-1}x\,dx = \int \frac{1}{1-x^2}\,dx$$

$$\tanh^{-1}x = \int \frac{1}{1-x^2}\,dx = \int 1 + x^2 + x^4 + x^6 + \dots\,dx$$

$$= x + \frac{1}{3}x^3 + \frac{1}{5}x^5 + \frac{1}{7}x^7 + \dots$$

5. Let $F(x) = \int_0^x \frac{\sin t\,dt}{t}$. Show that

$$F(x) = x - \frac{x^3}{3 \cdot 3!} + \frac{x^5}{5 \cdot 5!} - \frac{x^7}{7 \cdot 7!} + \ldots$$

Evaluate $F(1)$ to three decimal places.

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots$$

$$\frac{\sin x}{x} = 1 - \frac{x^2}{3!} + \frac{x^4}{5!} - \frac{x^6}{7!} + \ldots$$

$$F(x) = \int_0^x \frac{\sin t}{t}\,dt = \int_0^x 1 - \frac{t^2}{3!} + \frac{t^4}{5!} - \frac{t^6}{7!} + \ldots\,dx$$

$$= t - \frac{t^3}{3 \cdot 3!} + \frac{t^5}{5 \cdot 5!} - \frac{t^7}{7 \cdot 7!} + \ldots \Big|_0^x$$

$$= x - \frac{x^3}{3 \cdot 3!} + \frac{x^5}{5 \cdot 5!} - \frac{x^7}{7 \cdot 7!} + \ldots$$

$$F(1) \approx 1 - \frac{1}{3 \cdot 3!} + \frac{1}{5 \cdot 5!} - \frac{1}{7 \cdot 7!}$$

$$= 0.94608$$

$$\boxed{F(1) \approx 0.946}$$

6. Express the definite integral below as an infinite series and find its value. (Expand the infinite series to 4 terms to find the value.)

$$\int_0^1 \cos(x^2)dx$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots$$

$$\cos x^2 = 1 - \frac{x^4}{2!} + \frac{x^8}{4!} - \frac{x^{12}}{6!} + \ldots$$

$$\int_0^1 \cos x^2 \approx \int_0^1 \left(1 - \frac{x^4}{2!} + \frac{x^8}{4!} - \frac{x^{12}}{6!}\right) dx$$

$$= x - \frac{x^5}{5 \cdot 2!} + \frac{x^9}{9 \cdot 4!} - \frac{x^{13}}{13 \cdot 6!} \Big|_0^1$$

$$= \left(1 - \frac{1}{10} + \frac{1}{9 \cdot 24} - \frac{1}{13 \cdot 6!}\right) - (0)$$

$$\approx 0.904522792$$