

1 Arithmetic with Star (*) Operations Oct 2021 (No Calculators)

3 pts 1. If $x * y = 4x^4 + 3x^3 - (2y^3 + y^4)$, find the value of $2 * 3$.

Ans. _____

4 pts 2. Let $A * B = -3A + 2B$ and $A \# B = 13A - B$. Find X, if $(7 * X) * (X \# 2) = 19X$.

Ans. _____

5 Pts 3. Let $X * Y = X^2 + XY + Y^2$. Find the product ab of positive integers a and b, if $a * b = 37$.

Ans. _____

Solutions – Arithmetic with Star * Operations

- $2 * 3 = 4(2)^4 + 3(2)^3 - (2(3)^3 + (3)^4) = 4 \cdot 16 + 3 \cdot 8 - (2 \cdot 27 + 81) = 64 + 24 - (54 + 81) = 88 - 135$.
Ans. -47
- $7 * x = -21 + 2x$, $x * 2 = 13x - 2$, $(-21 + 2x) * (13x - 2) = 63 - 6x + 26x - 4 = 19x$,
 $20x + 59 = 19x$, $x = -59$.
Ans. -59
- Since the sum of the 2 integers is 37, the most that either could be is 6. By trial and error, we find that 3 and 4 will yield $9 + 12 + 16 = 37$. $3(4) = 12$.
Ans. 12

1 Arithmetic with “*” Operations Oct 2020 (No Calculators)

3 pts 1. Let $A * B = AB - B$. Find the value of $(10 * 11) - (11 * 10)$.

Ans. _____

4 pts 2. Let $m @ n$ be defined as $m^3n^2 - 5m^2n + mn^3$. Find the value of $(1 @ 2) @ 3$.

Ans. _____

5 pts 3. Let A be the greatest prime number less than $x^2y + y^2$. Let B be the least prime number greater than $xy^2 - x^2$. If $A * B = (A - B)(A + B)$, find $A * B$ when $x = 2$ and $y = 3$.

Ans. _____

Solutions Arithmetic with “ * ” Operations Oct 2020

1. $AB - B = (A - 1)B$. $10 * 11 = 9(11) = 99$. $11 * 10 = 10(10) = 100$. $99 - 100 = -1$. **Ans. -1**

2. $1 @ 2 = 1^3 \cdot 2^2 - 5 \cdot 1^2 \cdot 2 + 1 \cdot 2^3 = 4 - 10 + 8 = 2$. $2 @ 3 = 2^3 \cdot 3^2 - 5 \cdot 2^2 \cdot 3 + 2 \cdot 3^3 = 72 - 60 + 54 = 66$.

Ans. 66

3. $A \rightarrow 2^2 \cdot 3 + 3^2 = 12 + 9 = 21$; greatest prime less than 21 is 19. $B \rightarrow 2 \cdot 3^2 - 2^2 = 18 - 4 = 14$; least prime greater than 14 = 17. $A * B = (A - B)(A + B) = A^2 - B^2 \rightarrow 19^2 - 17^2 =$

$361 - 289 = 72$.

Ans. 72

1 Arithmetic with "*" Operations Oct 2019 (No Calculators)

3 pts 1. Let $x * y$ be defined as $2x^2 - 4xy + y^2$. Find the value of $(2 * 5) * 3$.

Ans. _____

4 pts 2. Find the value of $a + b$, when the 3-digit number $2a3$ is added to 326 to give the 3-digit number $5b9$, where $5b9$ is divisible by 9.

Ans. _____

5 pts 3. Let $a * b = \frac{2a^2 - b}{b - 2a^2}$ and $m @ n = \frac{4m - n^3}{m^3 - 4n}$. Find the value of $\frac{4 * 2}{2 @ 4}$.

Ans. _____

Solutions - Arithmetic with (*) Operations

1. $2 * 5 = 2(2)^2 - 4(2)(5) + 5^2 = 8 - 40 + 25 = -7$. $-7 * 3 = 2(-7)^2 - 4(-7)(3) + 3^2 = 98 + 84 + 9 = 191$.

Ans. 191

2. $2a3 + 326 = 5b9$. b has to be 4, so a is 2. $a + b = 2 + 4 = 6$.

Ans. 6

3. $4 * 2 = \frac{2(16) - 2}{2 - 2(16)} = \frac{30}{-30} = -1$. $2 @ 4 = \frac{4(2) - (4)^3}{2^3 - 4(4)} = \frac{8 - 64}{8 - 16} = \frac{-56}{-8} = 7$. $-1 \cdot \frac{1}{7} = -\frac{1}{7}$.

Ans. $-\frac{1}{7}$

1 Arithmetic with “*” Operations Oct 2018 (No Calculators)

3 pts 1. If $a * b = 2ab^2$, for what value of x is $x * 2x = 64$?

Ans. _____

4 pts 2. Measurements of the lengths of boards are recorded as whole numbers I and F , where I = the number of inches and F = the number of sixteenths of an inch. $0 \leq F \leq 15$. If the “star operation” expression to convert these measurements to feet is $I * F = \frac{I}{a} + \frac{F}{b}$, find ab .

Ans. _____

5 pts 3. Becca can mow a lawn in a certain number of minutes. It takes Geoff $\frac{2}{3}$ of the time it takes Becca to mow the same lawn. If they work together on their own respective mowers and Becca has a 5 minute head start, how long will it take to finish mowing the lawn? It takes Geoff 50 minutes to mow the lawn by himself.

Ans. _____

Solutions – Arithmetic with “*” Operations

1. $x * 2x = 2x(4x^2) = 8x^3 = 64$, so $x = 2$.

Ans. 2

2. $a = 12$, since there are 12 inches per foot. $b = 12(16) = 192$. $ab = 12(192) =$

Ans. 2304

3. $\frac{x+5}{75} + \frac{x}{50} = 1 \Rightarrow 2(x+5) + 3x = 150 \Rightarrow 2x + 10 + 3x = 150 \Rightarrow 5x = 140$.

Ans. 28 min

1 Arithmetic with Star Operations Oct 2017 (No Calculators)

3 pts 1. If $a * b = 2b - a^2$, find the value of $(3 * 4) - (4 * 3)$.

Ans. _____

4 pts 2. If $m \Delta n = m^n - n(n - m) + (m - n)^n$, find the value of $5 \Delta 2$.

Ans. _____

5 pts 3. If $p * q = (p - q)(p + q)$ and $p \Delta q = (p + q)^2 - 2pq$, find $(3 * 4)(3 \Delta 4)$.

Ans. _____

Solutions – Arithmetic with Star Operations

1. $[2(4) - 3^2] - [2(3) - 4^2] = (8 - 9) - (6 - 16) = -1 - (-10) = 9$

Ans. 9

2. $5^2 - 2(2 - 5) + (5 - 2)^2 = 25 - 2(-3) + (-3)^2 = 25 + 6 + 9 = 40$

Ans. 40

3. $(p - q)(p + q) = p^2 - q^2$. $(p + q)^2 - 2pq = p^2 + 2pq + q^2 - 2pq = p^2 + q^2$. Thus

$(p * q)(p \Delta q) = (p^2 - q^2)(p^2 + q^2)$, so $(3^2 - 4^2)(3^2 + 4^2) = 3^4 - 4^4 = 81 - 256 = -175$. **Ans. -175**

1 Arithmetic with “*” Operations Oct 2016-2017

3 pts 1. If $m \oslash n = m^2 - 6mn + 3n^3$, find the value of $-2 \oslash 3$.

Ans. _____

4 pts 2. Given $a \square b = a^2b - b^2a$ and $a \Delta b = 2a + 3b$, find the value of

$$(-3 \Delta 5) \square (4 \Delta -3)$$

Ans. _____

5 pts 3. The sum of three numbers is 98. The first number is $\frac{2}{3}$ of the second and the second number is $\frac{5}{8}$ of the third. What is the second number?

Ans. _____

Solutions – Arithmetic with “*” Operations

1. $-2 \oslash 3 = (-2)^2 - 6(-2)(3) + 3(3)^3 = 4 + 36 + 81 = 121$

Ans. 121

2. $-3 \Delta 5 = -6 + 15 = 9$. $4 \Delta -3 = 8 - 9 = -1$. $9 \square -1 = 9^2(-1) - (-1)^2(9) = -81 - 9$ Ans. -90

3. Let $b =$ second number: $\frac{2}{3}b + b + \frac{8}{5}b = 98 \rightarrow 10b + 15b + 24b = 98(15) \rightarrow$

$49b = 98(15)$, therefore $b = 30$.

Ans. 30

1 Arithmetic with Star Operations Oct 2015 (No Calculators)

3 pts 1. Let $a * b$ be defined as $a * b = 7a - 4b$. Find the value of $(4 * -7) * -2$.

Ans. _____

4 pts 2. The price of a can of Pepsi is fifty cents plus half its price. What would 2000 cans of Pepsi cost, in dollars?

Ans. _____

5 pts 3. Let $A * B$ be defined as $A * B = A^2 - 3AB + B^2$. Find the value of

$$(5 * (4 * (2 * 1))) + (4 * (5 * (1 * 2)))$$

Ans. _____

Solutions – Arithmetic with “*” Operations

1. $(4 * -7) = 7(4) - 4(-7) = 56$. $(56 * -2) = 7(56) - 4(-2) = 400$. **Ans. 400**

2. Let $x =$ price of can of Pepsi. Then $.5 + x/2 = x$. $x = 1$. 2000 cans cost **Ans. \$2000**

3. $2 * 1 = 4 - 3(2)(1) + 1 = -1 \rightarrow 4 * -1 = 16 - 3(4)(-1) + 1 = 29 \rightarrow$
 $5 * 29 = 25 - 3(5)(29) + 29^2 = 25 - 435 + 841 = 431$.
 $1 * 2 = 1 - 3(1)(2) + 4 = -1 \rightarrow 5 * -1 = 25 - 3(5)(-1) + 1 = 41 \rightarrow$
 $4 * 41 = 16 - 3(4)(41) + 41^2 = 16 - 492 + 1681 = 1205$. $431 + 1205 =$ **Ans. 1636**

1 Arithmetic with "*" Operations Oct 2014 (No Calculators)

3 pts 1. Let $\overline{\overline{X}} = X^2 - 1$. Find all possible values of A , if $\overline{\overline{A}} = 143$.

Ans. _____

4 pts 2. If $P * T = \left\lfloor \frac{(P-T) - \lfloor P-T \rfloor}{.25} \right\rfloor$, find $30.00 * 24.17$. Note that $\lfloor x \rfloor$ is the greatest integer less than or equal to x .

Ans. _____

5 pts 3. Given $a * b = 2a + 2b + 1$, $a \Delta b = a - b$ and $a \Theta b = a + b$, find $b - a$, if

$$a * b = a \Delta b \text{ and } a \Theta b = 243.$$

Ans. _____

Solutions – Arithmetic with * Operations

1. $\overline{\overline{X}} = a^2 - 1 = 143 \rightarrow a^2 = 144$, so $a = \pm 12$

Ans. ± 12

2. $\left\lfloor \frac{(30.00 - 24.17) - \lfloor 30.00 - 24.17 \rfloor}{.25} \right\rfloor = \left\lfloor \frac{5.83 - 5.00}{.25} \right\rfloor = \left\lfloor \frac{.83}{.25} \right\rfloor = 3$.

Ans. 3

3. $a * b = a \Delta b \rightarrow 2a + 2b + 1 = a - b \rightarrow (1) a + 3b = -1$; $(2) a + b = 243$. $(2) - (1) \rightarrow 2b = -244$, so $b = -122$. Thus $a = 365$. $b - a = -122 - 365 = -487$.

Ans. -487

1 Arithmetic with “*” Operations Oct 2013 (No Calculators)

3 pts 1. If $x * y = \frac{x+y}{y}$, determine the value of $2 * (3 * 4)$. Express your answer as a mixed number.

Ans. _____

4 pts 2. Find the value of the following in simplest form:

$$\left[\left(\frac{2}{5} \right)^{-1} + \left(\frac{3}{8} \right)^{-1} \right]^{-1}$$

Ans. _____

5 pts 3. If $x * y = x^2 - y^2$ and $x \# y = y^2 - x^2$. Find $(a * b) \# (a * c)$ in factored form.

Ans. _____

Answer Sheet – Arithmetic with “*” Operations

1. $3 * 4 = \frac{3+4}{4} = \frac{7}{4}$. $2 * \frac{7}{4} = \frac{2+7/4}{7/4} = \frac{8+7}{7} = \frac{15}{7}$. Ans. $2 \frac{1}{7}$

2. $\left(\frac{5}{2} + \frac{8}{3} \right)^{-1} = \left(\frac{31}{6} \right)^{-1} = \frac{6}{31}$. Ans. $6/31$

3. $(a^2 - b^2) \# (a^2 - c^2) = (a^2 - c^2)^2 - (a^2 - b^2)^2 = [(a^2 - c^2) - (a^2 - b^2)][(a^2 - c^2) + (a^2 - b^2)] =$
 $[a^2 - c^2 - a^2 + b^2][a^2 - c^2 + a^2 - b^2] = (b^2 - c^2)(2a^2 - b^2 - c^2)$ Ans. $(b-c)(b+c)(2a^2 - b^2 - c^2)$