2 Inequalities and Absolute Value Oct 2021 (No Calculators)

3 pts 1. Solve for x: $|x| = |x-8|$.

Ans. _____

4 pts 2. For $|x| \le 8$, the graph of the equation $|x| = |y|$ looks like the letter "X". What alphabetical letter does the graph $y = 2||x|-4|$ look like on the domain $|x| \le 8$?

Ans. _____

5 pts 3. For how many integral values of x, such that $|x| \le 4$, is the following inequality both defined and true?

$$\frac{x-3}{3-x} \le \frac{4}{3x+1}$$

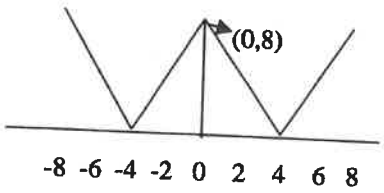
Ans. _____

Inequalities and Absolute Values

1. Either (1): x = x - 8 or (2): x = -(x – 8). (1): no solution. (2): x = 4. **Ans. 4**

2. Plotting:

x:	-8	-6	-4	-2	0	2	4	6	8
y:	8	4	0	4	8	4	0	4	8



Ans. W

3. The left hand side will always be -1 except at 3, which cannot be used. Because $|x| \le 4$, then the integers that can be used are from 4 to -4, 9 integers. From 0 to 4 will produce a positive number on the right side, all except 3 are OK. For -1, $-1 \le -2$ is not true, all other negative integers will work. 9 total minus 2 equals 7 integers that will make it true. **Ans. 7**

2 Inequalities and Absolute Values Oct 2020 (No Calculators)

3 pts 1. The equation $|3x-7| = 33$ has two solutions, A and B. Find the value of $|A| + |B|$.

Ans. _____

4 pts 2. Determine the smallest integral value m, so that $|5m-40| < 3m + 8$.

Ans. _____

5 pts 3. Find the sum of the four integers, closest to zero, that satisfy the following inequality

$$\frac{1}{3x-8} < \frac{2}{5x-1}.$$

Ans. _____

Inequalities and Absolute Values

1. (1): 3x – 7 = 33 or (2): 3x – 7 = -33. In (1): 3x = 40, $x = 13\frac{1}{3}$. In (2): 3x = -26, $x = -8\frac{2}{3}$.

$\left|13\frac{1}{3}\right| + \left|8\frac{2}{3}\right| = 22.$ **Ans: 22**

2. Critical points are at (1): 5m – 40 = 3m + 8, 2m = 48, m = 24; and (2) -5m + 40 = 3m + 8, -8m = -32, m = 4. Plugging in interval points: 0, 40 < 8, no; 10, 10 < 38, yes; 25, 85 < 83, no. Thus 4 < m < 24. Smallest integral value of m is 5. **Ans. 5**

3. Critical points are at $2\frac{2}{3}$, $\frac{1}{5}$ and where 5x -1 = 6x -16 or x = 15. Plugging in interval points; 0 ➔ $-\frac{1}{8} < -2$, no. 1➔ $-\frac{1}{5} <$, yes. 3 ➔ $1 < \frac{1}{7}$, no. 16 ➔ $\frac{1}{40} < \frac{2}{79}$ or 79 < 80, yes. Thus $\frac{1}{5} < x < 2\frac{2}{3}$ or x > 15. The 4 integers closest to zero are 1, 2, 16, 17. Their sum is 36. **Ans. 36**

3 pts 1. Find the sum of the solutions for $|2x-3|=7$. **Ans.** _____

4 pts 2. Find the values of x that satisfy the following:

$$|3x+4-(12x+2)|<3$$

Ans. _____

5 pts 3. Find the solutions for: $\frac{3x+1}{x-1}+\frac{1}{x+1}>3$.

Ans. _____

Inequalities and Absolute Values

1. $2x - 3 = 7 \rightarrow 2x = 10$, so $x = 5$. $2x - 3 = -7 \rightarrow 2x = -4$, so $x = -2$. $5 + (-2) = 3$. **Ans. 3**

2. $|3x+4-(12x+2)|<3 \rightarrow |-9x+2|<3$. Cr. pts. (1) $-9x + 2 = 3$, $x = -\frac{1}{9}$; (2) $-9x + 2 = -3$ $x = \frac{5}{9}$.

Checking: $-1 \rightarrow 11 < 3$, no; $0 \rightarrow 2 < 3$, yes; $1 \rightarrow 7 < 3$, no. **Ans.** $-\frac{1}{9} < x < \frac{5}{9}$

3. Cr. Pts are at -1 and 1 and where $(3x + 1)(x + 1) + x - 1 = 3(x^2 - 1) \rightarrow 5x = -3$ or $x = -3/5$.

Plugging in: $-2 \rightarrow 5/3 - 1 > 3$, no; $-4/5 \rightarrow 7/9 + 5 > 3$, yes; $0 \rightarrow -1 + 1 > 3$, no;

$2 \rightarrow 7 + 1/3 > 3$, yes. **Ans. $-1 < x < -3/5$ or $x > 1$**

2 Inequalities and Absolute Values Oct 2018 (No Calculators)

3 pts 1. How many integer values of x satisfy the inequality $|x| \le 5$?

Ans. _____

4 pts 2. The solution to the inequality $\frac{1}{|5-x|} > 16$ is "$a < x < b$, except $x \neq 5$". Find $b - a$.

Ans. _____

5 pts 3. In trying to solve $\frac{4}{x-2} > 5$, a student mistakenly uses a digit other than 5 when typing on his calculator, so he gets a wrong answer. If the answer he got was $2 < x < 4$, what digit did he use in place of the 5?

Ans. _____

Inequalities and Absolute Values

1. $|x| = 5$, so $-5 \le x \le 5$. Thus there are 11 integer values that work. **Ans. 11**

2. If $x > 5$, then $\frac{1}{x-5} > 16$, $1 > 16x - 80$, so $x < 81/16$. If $x < 5$, then $\frac{1}{5-x} < 16$, $1 > 80 - 16x$, so $x > 79/16$. $b - a = 81/16 - 79/16 = 2/16 = 1/8$. **Ans. 1/8**

3. $2 < x < 4 \rightarrow 2 - 2 < x - 2 < 4 - 2$, so $0 < x - 2 < 4$, thus $\frac{4}{x-2} < \frac{4}{2}$ or $\frac{4}{x-2} < 2$. **Ans. 2**

2 Inequalities and Absolute Values Oct 2017 (No Calculators)

3 pts 1. Find the values of x such that $|3x-5|<15+7x$.

Ans. _____

4 pts 2. Find all values for x which satisfy the following:

$$(x-2)(x-5)\,(x-2)(x+2)<0$$

Ans. _____

5 pts 1. Find all possible values of x, such that:

$$\frac{5}{x-3}\geq\frac{2}{2x+1} \text{ and } x^2-7x-30\leq 0.$$

Ans. _____

Inequalities and Absolute Values

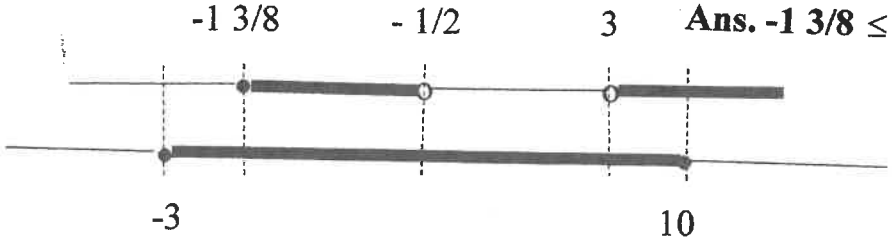
1. Critical points are where (1) 3x – 5 = 15 + 7x and (2) 3x – 5 = -15 -7x. In (1) -20 = 4x, thus x = -5. In (2) 10x = -10, thus x = -1. Plugging in interval points: -6 ➔ 23 < -27, no; -2 ➔ 11 < 1, no; 0 ➔ 5 < 15, yes. So x > -1.

Ans. x > -1

2. Critical points are at -2, 2, 5. Plugging in just interval points to see if the product is less than zero: -3 ➔ - - - - < 0, no; 0 ➔ - - - + < 0, yes; 3 ➔ + - + - < 0, yes; 6 ➔ + + + + < 0 , no.

Ans. -2 < x < 2 or 2 < x < 5, or -2 < x < 5, but x ≠ 2

3. For $\frac{5}{x-3}\geq\frac{2}{2x+1}$, critical points are at 3, - ½ and where $\frac{5}{x-3}=\frac{2}{2x+1}$: 10x +5 = 2x – 6, so 8x = -11 or x = -1 3/8. For interval point check: -2 ➔ -1 ≥ -2/3, no; -1 ➔ -1¼ ≥ -2, yes; 0 ➔ -1 2/3 ≥ 2, no; 4 ➔ 5 ≥ 2/9, yes. For $x^2-7x-30\leq 0$, (x – 10)(x + 3) ≥ 0. Cr pts 10, 3. For interval points: -4 ➔ - - ≤ 0, no; 0 ➔ - + ≤ 0, yes; 11 ➔ + + ≤ 0, no. On the expanded number line below:



Ans. -1 3/8 ≤ x < -1/2 or 3 < x ≤ 10

2 Inequalities and Absolute Values Oct 2016-2017

3 pts 1. Find the solution for the following: $3x - 8 \le 7$ and $8 - 4x \le 20$.

Ans. _____

4 pts 2. Find all x such that $x^3 - x \le 3 - 3x^2$.

Ans. _____

5 pts 3. Find the solution for x given that $3|4x-5| \le 6|x+2|$.

Ans. _____

Inequalities and Absolute Values

1. $3x - 8 \le 7$ ➔ $3x \le 15$, so $x \le 5$. $8 - 4x \le 20$ ➔ $-12 \le 4x$, so $x \ge -3$. **Ans. $-3 \le x \le 5$**

2. $x^3 - x \le 3 - 3x^2$ ➔ $x^3 + 3x^2 - x - 3 \le 0$ ➔ $(x^2 - 1)(x + 3) \le 0$ ➔ $(x + 1)(x - 1)(x + 3) \le 0$.

Critical points are at -3, -1, 1. Plugging in interval points into the last inequality, looking only for products of signed numbers: -4: (-)(-)(-) ≤ 0, yes; -2: (-)(-)(+) ≤ 0, no; 0: (+)(-)(+) ≤ 0, yes; 2: (+)(+)(+) ≤ 0, no.

Ans. $x \le -3$ or $-1 \le x \le 1$

3. Critical points are where (1) $3(4x - 5) = 6(x + 2)$ and where (2) $3(4x - 5) = -6(x + 2)$.

In (1): $12x - 15 = 6x + 12$ ➔ $6x = 27$, $x = 4\frac{1}{2}$. In (2): $12x - 15 = -6x - 12$ ➔ $18x = 3$, $x = 1/6$.

Plugging in interval points: 0: $15 \le 12$; no. 1: $3 \le 18$, yes. 5: $45 \le 42$, no. **Ans. $1/6 \le x \le 4\frac{1}{2}$**

3 pts 1. Simplify $\dfrac{|3-|2-6||+|3-5|^2}{|-2-8|}$

Ans. _____

4 pts 2. The solution for the inequality $\left|\dfrac{4x-8}{9}\right| \le 7$ can be written as $a \le x \le b$,

Find $a + b$.

Ans. _____

5 pts 3. For what values of x is the inequality $\dfrac{3x}{2x-4} \le \dfrac{1}{1-x}$ true?

Ans. _____

Inequalities and Absolute Values

1. $\dfrac{|3-4|+4}{10} = \dfrac{5}{10} = \dfrac{1}{2}$. **Ans. ½**

2. $-7 \le \dfrac{4x-8}{9} \le 7 \Rightarrow -63 \le 4x-8 \le 63 \Rightarrow -55 \le 4x \le 71 \Rightarrow \dfrac{-55}{4} \le x \le \dfrac{71}{4}$. a + b = **Ans. 4**

3. Critical points are at 2 and 1 and where

$$\frac{3x}{2x-4} = \frac{1}{1-x} \Rightarrow 3x-3x^2 = 2x-4 \Rightarrow 3x^2-x-4=0 \Rightarrow (3x-4)(x+1)=0,\ x = 4/3 \text{ or } -1.$$

Plugging in interval points: -2 ➔ $\dfrac{-6}{-8} \le \dfrac{1}{3}$, no; 0 ➔ $0 \le 1$, yes;

7/6 ➔ $\dfrac{3.5}{-5/3} \le -6$, no; 5/3 ➔ $\dfrac{5}{-2/3} \le \dfrac{1}{-2/3}$, yes.

3 ➔ $\dfrac{9}{2} \le \dfrac{1}{-2}$, no.

Ans. $-1 \le x < 1$ or $4/3 \le x < 2$

2 Inequalities and Absolute Values Oct 2014 (No Calculators)

3 pts 1. For how many integers is $8 - |4 - x| > 0$?

Ans. _____

4 pts 2. Find all values of x such that $|1 - |2 - |3 - x|| = 0$?

Ans. _____

5 pts 3. For $x \neq 0$, solve $\frac{20}{x} > |10 - x|$.

Ans. _____

Inequalities and absolute values

1. $8 - |4 - x| > 0 \rightarrow |4 - x| < 8 \rightarrow$ CP: $4 - x = 8$, $x = -4$; $4 - x = -8$, $x = 12$. So all integers from -3 to 11. There are 15. **Ans. 15**

2. $|2 - |3 - x||$ must = 1. Thus (1) $2 - |3 - x| = 1$ or (2) $2 - |3 - x| = -1$. In (1) $3 - x = \pm 1 \rightarrow$ $x = 2$ or 4. In (2) $3 - x = \pm 3 \rightarrow x = 0$ or 6. **Ans. 0, 2, 4, 6**

3. (1) Since right side ≥ 0, x must be greater than 0. (2) For $0 < x \leq 10$, $\frac{20}{x} > 10 - x \rightarrow$

CP: $x^2 - 10x + 20 > 0$. $x = \frac{10 \pm \sqrt{20}}{5} = 5 \pm \sqrt{5} \rightarrow 0 < x < 5 - \sqrt{5}$ or $5 + \sqrt{5} < x \leq 10$.

(3) For $x > 10$, $\frac{20}{x} > x - 10 \rightarrow x^2 - 10x - 20 < 0 \rightarrow$ CP: $x = \frac{10 \pm \sqrt{180}}{2} = 5 \pm 3\sqrt{5}$.

So $10 < x < 5 + 3\sqrt{5}$.

Using just critical points: $5 - \sqrt{5} = 2.5$, $5 + \sqrt{5} = 7.5$, $5 - 3\sqrt{5} = -1.5$, $5 + 3\sqrt{5}$ and $x = 0$.
These just approximations, and the number line: ---o-------o━━━━o-------o━━━━o----

-1.5 0 2.5 7.5 11.5

Plugging in interval points: -2 → $-10 > 12$, no; -1 → $-20 > 11$, no; 1 → $20 > 9$, yes; 4 → $5 > 6$, no; 10 → $2 > 0$, yes; 20 → $1 > 10$, no.

Ans. $0 < x < 5 - \sqrt{5}$ or $5 + \sqrt{5} < x < 5 + 3\sqrt{5}$

2 Inequalities and Absolute Values Oct 2013 (No Calculators)

3 pts 1. Find the solutions to the following:

$$|5-x| = |2x|$$

Ans. _____

4 pts 2. x is an even integer. Find the largest such x for which:

$$2x^2 - 5 \le -9x$$

Ans. _____

5 pts 3. Find the values of x such that:

$$|x^2-1| - |x| > |x+1|$$

Ans. _____

Inequalities and Absolute Values

1. (1) $5 - x = 2x$ or (2) $5 - x = -2x$. In (1): $5 = 3x$ or $x = 5/3$. In (2): $x = -5$. **Ans. 5/3, -5**

2. $2x^2 - 5 \le -9x \rightarrow 2x^2 + 9x - 5 \le 0 \rightarrow (2x-1)(x+5) \le 0 \rightarrow -5 \le x \le ½$. **Ans. 0**

3. CP: $x = 1, x = 0, x = -1$. For $x \ge 1$: $x^2 - 1 - x > x + 1 \rightarrow x^2 - 2x - 2 > 0 \rightarrow$ CP: $x = 1 \pm \sqrt{3}, x > 1 + \sqrt{3}$. For $0 \le x < 1$: $-x^2 + 1 - x > x + 1 \rightarrow -x^2 - 2x > 0 \rightarrow$ $x(x+2) < 0 \rightarrow -2 < x < 0$. No solution. For $-1 \le x < 0$: $-x^2 + 1 + x > x + 1 \rightarrow -x^2 > 0$, $x^2 < 0$. No solution. For $x < -1$: $x^2 - 1 + x > -x - 1 \rightarrow x^2 + 2x > 0 \rightarrow x(x+2) > 0$, $x < -2$. **Ans. $x > 1 + \sqrt{3}$ or $x < -2$**