3 Matrices, Determinants & Systems of Equations Oct 2021 (No Calculators)

3 pts 1. Solve for x: $\begin{vmatrix} -4 & 3 \\ 7 & x \end{vmatrix} = \begin{vmatrix} 2 & x \\ -1 & -5 \end{vmatrix}$

Ans. ____________________

4 pts 2. If $A = \begin{bmatrix} -2 & 5 \\ 3 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 7 \\ 0 & -3 \end{bmatrix}$, find determinant of $B^{-1} \cdot A^{-1}$.

Ans. ____________________

5 pts 3. Find the value of a + b + c, if

3a – b + 6c = -2

2a + 2b + c = -8

-4a – 3b + 4c = 4

Ans. ____________________

Matrices, Determinants and Systems of Equations

1. – 4x – 21 = -10 + x ➔ -5x = 11, x = -11/5 or $-2\frac{1}{5}$. **Ans. -11/5** or $-2\frac{1}{5}$

2. $B^{-1} \cdot A^{-1} = (AB)^{-1} = \begin{bmatrix} -2 & -29 \\ 3 & 24 \end{bmatrix}^{-1} = \frac{1}{39}\begin{bmatrix} 24 & 29 \\ -3 & -2 \end{bmatrix} = \begin{bmatrix} \frac{8}{13} & \frac{29}{39} \\ \frac{-1}{13} & \frac{-2}{39} \end{bmatrix}$. The determinant of this is $\frac{-16}{13(39)} + \frac{29}{13(39)} = \frac{13}{13(39)} = \frac{1}{39}$. **Ans. 1/39**

3. From the question, 2(1) + (2) ➔ (4) 8a + 13c = -12; -3(1) + 3 ➔ (5) -13a + 14c = 10.
13(4): 104a + 169c = -156 8(5): -104a – 112c = 80. Adding these: 57c = -76 or c = -4/3.
Using (4): 8a – 52/3 = -12, 8a = 16/3, so a = 2/3. Using (1): 3(2/3) – b + 6(-4/3) = -2 ➔
2 – b – 8 = -2, -4 = b. a + b + c = 2/3 + (-4) + (-4/3) = $-4\frac{2}{3}$. **Ans.** $-4\frac{2}{3}$

3 Matrices, Determinants and Linear Systems Oct 2020 (No Calculators)

3 pts 1. Solve the following for x: $\begin{vmatrix} 3x & 7 \\ 5x & 12 \end{vmatrix} = 72.$

Ans. ______________

4 pts 2. Find $|AB|$, if A = $\begin{bmatrix} 3 & 2 & 1 \\ 1 & 5 & -4 \end{bmatrix}$ and B = $\begin{bmatrix} 4 & 2 \\ -5 & 7 \\ 3 & 1 \end{bmatrix}$.

Ans. ______________

5 pts 3. Find the value of 5a, if

$$2a + 3b + 5c = 59$$
$$3a + 5b + 2c = 59$$
$$5a + 2b + 3c = 52$$

Ans. ______________

Matrices, Determinants and Linear Systems

1. 36x – 35x = 72, so x = 72. **Ans. 72**

2. AB = $\begin{bmatrix} 3 & 2 & 1 \\ 1 & 5 & -4 \end{bmatrix}\begin{bmatrix} 4 & 2 \\ -5 & 7 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} 12-10+3 & 6+14+1 \\ 4-25-12 & 2+35-4 \end{bmatrix} = \begin{bmatrix} 5 & 21 \\ -33 & 33 \end{bmatrix}$. $|AB| = 5(33) + 21(33) =$ 26(33) = 858. **Ans. 858**

3. Adding all 3: 10a + 10b + 10 c = 170, or a + b + c = 17. -2a – 2b -2c = -34 added to the first equation 2a + 3b + 5c = 59 yields (1) b + 3c = 25. Adding – 3a – 3b – 3c = -51 to th second equation yields (2) 2b – c = 8. -2(1) + (2): (-2b – 6c = -50) + (2b - c = 8) yields -7c = -42, so c = 6. In (2): 2b – 6 = 8, 2b = 14, so b = 7. A + (7) + (6) = 17, so a = 4. 5(4) = 20. **Ans. 20**

3 Matrices, Determinants and Linear Systems Oct 2019 (No Calculators)

3 pts 1. What is the y-coordinate of the point at which the lines y = 20x + 19 and y = 19x + 20 intersect?

Ans. ____________

4 pts 2. Given that AB = C, find the value of $a + b + c$, if

$$A = \begin{bmatrix} a & 2 & 5 \\ 0 & 1 & 4 \\ 2 & 3 & 6 \end{bmatrix}, B = \begin{bmatrix} 1 & 5 & 3 \\ 2 & b & 7 \\ 1 & 0 & 5 \end{bmatrix} \text{ and } C = \begin{bmatrix} 10 & 7 & 42 \\ 6 & 1 & 27 \\ 14 & 13 & c \end{bmatrix}.$$

Ans. ____________

5 pts 3. Solve the following for x:

$$\begin{vmatrix} 1 & x & 4 \\ 3 & 0 & -2 \\ 1 & -1 & 3 \end{vmatrix} = \begin{vmatrix} 4 & 0 & 2 \\ x & 1 & -1 \\ 0 & 3 & 2 \end{vmatrix}.$$

Ans. ____________

Matrices, Determinants and Linear Systems

1. 20x + 19 = 19x + 20, x = 1. So y = 20(1) + 19 = 39. **Ans. 39**

2. $A_{R1}(B_{C1})$ = a + 4 + 5 = 10, so a = 1. $A_{R2}(B_{C2})$ = 0 + b + 0 = 1, s0 b = 1. $A_{R3}(B_{C3})$ =

6 + 21 + 30 = = c, so = 57. a + b + c = 1 + 1 + 57 = 59. **Ans. 59**

3. $\begin{vmatrix} 1 & x & 4 \\ 3 & 0 & -2 \\ 1 & -1 & 3 \end{vmatrix} = \begin{vmatrix} 4 & 0 & 2 \\ x & 1 & -1 \\ 0 & 3 & 2 \end{vmatrix}$ → 0 - 2x – 12 + 0 – 2 – 9x = 8 + 0 + 6x – 0 +12 + 0 →

- 11x – 14 = 6x + 20, x = - 2. **Ans. -2**

3 Matrices, Determinants and Systems of Equations Oct 2018 (No Calculators)

3 pts 1. Apples and bananas are priced by the piece. If 4 apples and 1 banana cost 28 cents less than 2 apples and 6 bananas while 2 apples and 3 bananas cost 3 cents less than 3 apples and 1 banana, how many cents less do 3 apples cost than 7 bananas?

Ans. ____________________

4 pts 2. The matrix $\begin{bmatrix} 1 & 3 & -5 \\ -2 & 0 & 2 \end{bmatrix}$ can be the factor on the left in a matrix multiplication only when one particular matrix below is the factor on the right. Perform this multiplication and give the sum of the elements in the product matrix for your answer.

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $\quad \begin{bmatrix} 5 & 0 & 2 \end{bmatrix}$ $\quad \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$ $\quad \begin{bmatrix} -5 & 3 & 1 \\ 2 & 0 & -2 \end{bmatrix}$

Ans. ____________________

5 pts 3. Find the quotient $\dfrac{\begin{bmatrix} 5 & 2 \\ 3 & 4 \end{bmatrix}}{\begin{bmatrix} 6 & 8 \\ 5 & 7 \end{bmatrix}}$.

Ans. ____________________

Matrices, Determinants and Systems of Equations

1. 4a + b = 2a + 6b – 28 or (1): 2a – 5b = -28. 2a + 3b = 3a + b – 3 or (2): –a + 2b = -3. (2) – (1): -3a + 7b = 25. Or, solving we get a = 71 and b = 34. 7b – 3a = 238 – 213 = 25. **Ans. 25**

2. $\begin{bmatrix} 1 & 3 & -5 \\ -2 & 0 & 2 \end{bmatrix}\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} -5 & -2 & -4 \\ 2 & 2 & 0 \end{bmatrix}$. Sum of elements = -7. **Ans. -7**

3. $\dfrac{\begin{bmatrix} 5 & 2 \\ 3 & 4 \end{bmatrix}}{\begin{bmatrix} 6 & 8 \\ 5 & 7 \end{bmatrix}} = \begin{bmatrix} 5 & 2 \\ 3 & 4 \end{bmatrix} \cdot \begin{bmatrix} 6 & 8 \\ 5 & 7 \end{bmatrix}^{-1}$. $\begin{bmatrix} 6 & 8 \\ 5 & 7 \end{bmatrix}^{-1} = \dfrac{1}{42-40}\begin{bmatrix} 7 & -8 \\ -5 & 6 \end{bmatrix} = \begin{bmatrix} 3\frac{1}{2} & -4 \\ -2\frac{1}{2} & 3 \end{bmatrix}$. Thus

$\begin{bmatrix} 5 & 2 \\ 3 & 4 \end{bmatrix} \cdot \begin{bmatrix} 3\frac{1}{2} & -4 \\ -2\frac{1}{2} & 3 \end{bmatrix} = \begin{bmatrix} 12\frac{1}{2} & -14 \\ \frac{1}{2} & 0 \end{bmatrix}$.

Ans. $\begin{bmatrix} 12\frac{1}{2} & -14 \\ \frac{1}{2} & 0 \end{bmatrix}$

3 Matrices, Determinants and Linear Systems Oct 2017 (No Calculators)

3 pts 1. Solve the following for x: $\begin{vmatrix} 3 & 5 \\ x-3 & x \end{vmatrix} = 7x - 66.$

Ans. ____________

4 pts 2. If $5x + 9y = -13$ and $4x + 11y = 1$, find the value of $3x - 6y$.

Ans. ____________

5 pts 3. Find the determinant of the matrix: $\begin{bmatrix} -3 & 2 & -5 & -1 \\ 2 & 4 & 6 & 3 \\ 1 & -2 & 3 & 4 \\ 3 & 1 & 9 & 2 \end{bmatrix}.$

Ans. ____________

Matrices, Determinants and Linear Systems

1. $3x - 5(x - 3) = 7x - 66 \Rightarrow -2x + 15 = 7x - 66 \Rightarrow 81 = 9x$, so $x = 9$. **Ans. 9**

2. (1): $5x + 9y = -13$, (2): $-4x + 11 = -1$; (1) + (2): $x - 2y = -14 \Rightarrow 3x - 6y = -42$. **Ans. -42**

3. $\begin{bmatrix} -3 & 2 & -5 & -1 \\ 2 & 4 & 6 & 3 \\ 1 & -2 & 3 & 4 \\ 3 & 1 & 9 & 2 \end{bmatrix}$, mult. column 1 by -3 and add to column 3: $\begin{bmatrix} -3 & 2 & 4 & -1 \\ 2 & 4 & 0 & 3 \\ 1 & -2 & 0 & 4 \\ 3 & 1 & 0 & 2 \end{bmatrix}.$

Using determinants by minors: $4\begin{vmatrix} 2 & 4 & 3 \\ 1 & -2 & 4 \\ 3 & 1 & 2 \end{vmatrix} = 4[(18 - 8 - 8) - (-8 + 48 + 3)] = 4(45)$. **Ans. 180**

3 Matrices, Determinants and Systems of Equations Oct 2016-2017

3 pts 1. Solve for x and y, if: $63x + 37y = 263$ and $37x + 63y = 237$. Express your answer in (x, y) form.

Ans. ______________________

4 pts 2. Solve for the matrix A, if: $\begin{bmatrix} 2 & -1 \\ -3 & 3 \end{bmatrix} A = \begin{bmatrix} 10 & -3 \\ -18 & 12 \end{bmatrix}$.

Ans. ______________________

5 pts 3. Solve the following system: Give answer in ordered pair, (x,y), form.

$$x^{1/4} + y^{1/5} = 5$$

$$x^{3/4} + y^{3/5} = 35$$

Ans. ______________________

Matrices, Determinants and Systems of Equations

1. Adding the two equations: $100x + 100y = 500$ or (1) $x + y = 5$. Subtracting the two: $26x - 26y = 26$ or (2) $x - y = 1$. Adding (1) and (2) $2x = 6$, $x = 3$. So $y = 2$. **Ans. (3, 2)**

2. The inverse of $A = \frac{1}{3}\begin{bmatrix} 3 & 1 \\ 3 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 1/3 \\ 1 & 2/3 \end{bmatrix}$. Multiplying both sides by the inverse, makes the right side $\begin{bmatrix} 1 & 1/3 \\ 1 & 2/3 \end{bmatrix}\begin{bmatrix} 10 & -3 \\ -18 & 12 \end{bmatrix} = \begin{bmatrix} 4 & 1 \\ -2 & 5 \end{bmatrix}$ **Ans.** $\begin{bmatrix} 4 & 1 \\ -2 & 5 \end{bmatrix}$

3. Let $x^{\frac{1}{4}} = a$ and $y^{\frac{1}{5}} = b$, then $a + b = 5$ and $a^3 + b^3 = 35$. $(a + b)^3 = 125$. Therefore $a^3 + 3a^2b + 3ab^2 + b^3 = 125 \rightarrow (a^3 + b^3) + 3ab(a+b) = 125 \rightarrow 35 + 3ab(15) = 125 \rightarrow$ $15ab = 90$ or $ab = 6$. Since $a + b = 5$, then either $a = 3$ and $b = 2$, or $a = 2$ and $b = 3$. If $x^{\frac{1}{4}} = 3$ when $y^{\frac{1}{5}} = 2$, then $x = 81$ and $y = 32$. If $x^{\frac{1}{4}} = 2$ when $y^{\frac{1}{5}} = 3$, then $x = 16$ and $y = 243$. **Ans. (81, 32) or (16, 243)**

Matrices, Determinants and Systems of Equations Oct 2015 (No Calculators)

3 pts 1. Find the sum of the values of A, B, C, and D given $\begin{bmatrix} A & A-B \\ B+3C & C-2D \end{bmatrix} = \begin{bmatrix} 4 & 10 \\ 18 & 0 \end{bmatrix}$.

Ans. ________________

4 pts 2. Given that $\frac{1}{x}+\frac{2}{y}=6$ and $\frac{4}{x}+\frac{9}{y}=5$, find the value of $\frac{1}{x}-\frac{1}{y}$.

Ans. ________________

5 pts 3. Let A = $\begin{bmatrix} 5 & 3 & 1 \\ 9 & 2 & 2 \\ 0 & x & y \end{bmatrix}$. If A• $\begin{bmatrix} 1 \\ 3 \\ 6 \end{bmatrix} = \begin{bmatrix} 20 \\ 27 \\ 33 \end{bmatrix}$ and the determinant of A = -71.

Find the product xy.

Ans. ________________

Matrices, Determinants and Linear Systems

1. A = 4. A –B = 10 ➔ 4 – B = 10, so B = -6. B + 3C = 18 ➔ -6 + 3C = 18, so C = 8. C – 2D = 0 ➔ 8 = 2D, so D = 4. A + B + C + D = 4 + (-6) + 8 + 4 = 10. **Ans. 10**

2. Let a = 1/x and b = 1/y, then (1) a + 2b = 6 and (2) 4a + 9b = 5. (2) – 4(1): b = -19. In (1): a + 2(-19) = 6, so a = 44. 1/x - 1/y = a – b = 44 – (-19) = 63. **Ans. 63**

3. $|A|$ = 10y + 9x - 10x – 27y = -71, or (1) –x – 17y = -71. In multiplying the matrices: Third row by column yields 3x + 6y = 33 or (2) x + 2y = 11. (1) + (2): -15y = -60, so y = 4. In (2): x + 2(4) = 11, so x = 3. Xy = (3)(4) = 12. **Ans. 12**

3 Matrices, Determinants, and Systems of Equations Oct 2014 (No Calculators)

3 pts 1. Find the determinant of $\begin{bmatrix} 2 & -1 \\ 0 & -4 \end{bmatrix}\begin{bmatrix} 3 & 2 \\ -2 & 0 \end{bmatrix}$

Ans. ________________

4 pts 2. Find a + b + c + d, if: $\begin{cases} a = b + c \\ b = c + d \\ c = a + 2 \\ d = a - b \end{cases}$

Ans. ________________

5 pts 3. There exists only one 2-by-2 matrix **A** such that $\mathbf{A}^{-1}\begin{bmatrix} -3 & 1 \\ 4 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -5 & 1 \end{bmatrix}$. Find **A**.

Ans. ________________

Matrices, Determinants and Systems of Equations

1. $\begin{bmatrix} 2 & -1 \\ 0 & -4 \end{bmatrix}\begin{bmatrix} 3 & 2 \\ -2 & 0 \end{bmatrix} = \begin{bmatrix} 8 & 4 \\ 8 & 0 \end{bmatrix} = -32$. **Ans. -32**

2. b = c + d = c + (a – b) = (b + c) + (c – b) = 2c. a = b + c = 2c + c = 3c.
c = a + 2 + 3c + 2 ➔ 2c = -2 ➔ c = 1, b = -2, a = -3. d = a – b = -3 – (-2) = -1.
a + b + c + d = (-3) + (-2) + (-1) + (-1) = -7. **Ans. -7**

3. Let $B = \begin{bmatrix} -3 & 1 \\ 4 & -1 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 0 \\ -5 & 1 \end{bmatrix}$. $A^{-1}B = C$ ➔ $AA^{-1}B = AC$ ➔ $B = AC$ ➔

$BC^{-1} = ACC^{-1}$ ➔ $BC^{-1} = A$. $C^{-1} = \frac{1}{1}\begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix}$. $BC^{-1} = \begin{bmatrix} -3 & 1 \\ 4 & -1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix} =$ **Ans.** $\begin{bmatrix} 2 & 1 \\ -1 & -1 \end{bmatrix}$

3 Matrices, Determinants and Systems of Equations Oct 2013 (No Calculators)

3 pts 1. Find the ordered pair (x, y), such that $4x + 3y = 5$ and $7y - 3x = 24$.

Ans. ____________

4 pts 2. Perform as indicated $\begin{bmatrix} 4 & -1 & 3 \\ -5 & 2 & 1 \end{bmatrix}\begin{bmatrix} -3 & 2 \\ 2 & 5 \\ 4 & -3 \end{bmatrix} + \begin{bmatrix} 5 & 7 \\ -23 & 5 \end{bmatrix}$

Ans. ____________

5 pts 3. Find the value of the following:

$$\begin{vmatrix} 1 & 2 & 3 & 0 \\ 2 & 1 & 0 & 0 \\ 2 & 0 & 1 & 5 \\ 1 & 1 & 1 & 1 \end{vmatrix}$$

Ans. ____________

Matrices, Determinants and Systems of Equations

1. (1) $3(4x + 3y = 5) = 12x + 9y = 15$, (2) $4(7y - 3x = 24) = 28y - 12x = 96$. Adding: $9y + 28y = 15 + 96 \Rightarrow 37y = 111$, $y = 3$. In (1): $4x + 3(3) = 5$, $x = -1$. **Ans. (-1, 3)**

2. $$\begin{bmatrix} 4 & -1 & 3 \\ -5 & 2 & 1 \end{bmatrix}\begin{bmatrix} -3 & 2 \\ 2 & 5 \\ 4 & -3 \end{bmatrix} + \begin{bmatrix} 5 & 7 \\ -23 & 5 \end{bmatrix} = \begin{bmatrix} -12-2+12 & 8-5-9 \\ 15+4+4 & -10+10-3 \end{bmatrix} = \begin{bmatrix} -2 & -6 \\ 23 & -3 \end{bmatrix} + \begin{bmatrix} 5 & 7 \\ -23 & 5 \end{bmatrix}$$

Ans. $\begin{bmatrix} 3 & 1 \\ 0 & 2 \end{bmatrix}$

3. $$\begin{vmatrix} 1 & 2 & 3 & 0 \\ 2 & 1 & 0 & 0 \\ 2 & 0 & 1 & 5 \\ 1 & 1 & 1 & 1 \end{vmatrix} = -2C_2 + C_1 = \begin{vmatrix} -3 & 2 & 3 & 0 \\ 0 & 1 & 0 & 0 \\ 2 & 0 & 1 & 5 \\ -1 & 1 & 1 & 1 \end{vmatrix} = \begin{vmatrix} -3 & 3 & 0 \\ 2 & 1 & 5 \\ -1 & 1 & 1 \end{vmatrix} = C_2 + C_1 = \begin{vmatrix} 0 & 3 & 0 \\ 3 & 1 & 5 \\ 0 & 1 & 1 \end{vmatrix} = -3\begin{vmatrix} 3 & 5 \\ 0 & 1 \end{vmatrix} =$$

$-3(3) = -9$. **Ans. -9**