## 4 Number Theory Oct 2021 (No Calculators)

**3 pts 1.** What Lowest Common Denominator (LCD) should be used to add the following fractions?

$$\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\frac{1}{6}+\frac{1}{7}+\frac{1}{8}+\frac{1}{9}+\frac{1}{10}$$

**Ans.** ____________________

**4 pts 2.** The product of two positives integers is 486,000. If the greatest common factor of the two integers is 90, what is the least common multiple of the two integers?

**Ans.** ____________________

**5 pts 3.** Find the sum of the 3 smallest positive integers such that when each is divided be 4 the remainder will be 1, when each is divided by 5 the remainder will be 2, and when each is divided by 6 the remainder will be 3.

**Ans.** ____________________

## Number Theory

**1.** The LCD of 2 through 10 is $2^3 \cdot 3^2 \cdot 5 \cdot 7 = 72(35) = 2520$. **Ans. 2520**

**2.** For two positive integers, product = GCF·LCM, LCM = $\frac{486{,}000}{90} = 5400$. **Ans. 5400**

**3.** Let N be the first such number. Then N = 4a + 1, N = 5b + 2 and N = 6c + 3.

4a + 1 = 5b + 2 ➔ 4a = 5b + 1. If b = 3, then a = 4 and N = 17. This is only for 4 and 5, now we have N = 20d + 17 to go with N = 6c + 3 ➔ 20d + 17 = 6c + 3, 6c = 20d + 14. When d = 2, c = 9. So N = 6(9) + 3 = 57. The second number will be 57 + 2(5)6 = 57 + 60 = 117. The third will be 117 + 60 = 117. 57 + 117 + 177 = 351. **Ans. 351**

## 4 Number Theory Oct 2020 (No Calculators)

**3 pts 1.** Find the sum of the prime numbers between 80 and 100.

**Ans.** ____________________

**4 pts 2.** Find the fifth whole number greater than zero which when divided by 8 has a remainder of 3 and when divided by 7 has a remainder of 2.

**Ans.** ____________________

**5 pts 3.** Find the unit's digit of the number of the following sum:

$$7^{87} + 8^{89} + 9^{91}$$

**Ans.** ____________________

## Number Theory

**1.** 83,89, 97 are the primes between 80 and 100. Their sum is 269. **Ans. 269**

**2.** Let the number be x. Then x = 8a + 3 and x = 7b + 2. Thus 8a + 3 = 7b + 2 or 7b = 8a + 1.

Plugging in 0 through 6 for a to find a number 7 will divide into. When a = 6, b = 7.

Thus x = 7(7) + 2 = 51. This is the first, 51 + 56(4) = the fifth = 51 + 224 = 275. **Ans. 275**

**3.** $7^1 = 7$, $7^2 = 49$, unit's digit is 9, third power is 3, 4th power is 1, 5th power is 7. So it repeats every 4 powers of 7. If you divide 87 by 4, the remainder is 3, so the unit's digit is 3. 8 has unit' digits of 8, 4, 2, 6, and back to 8 again for its powers. It also repeats every 4 powers and 89 divided by 4 leaves a remainder of 1, which would mean 8. The powers of nine are easier, for every odd power the unit's digit is 9 and for even powers it is 1. So since odd it is 9. Adding 3, 8, and 9 equals 20. So the unit's digit is 0. **Ans. 0**

## 4 Number Theory Oct 2019 (No Calculators)

**3 pts 1.** Convert the base six number 1234 to a base 10 number.

**Ans.** ____________________

**4 pts 2.** Two positive numbers x and y have a product of 768 and a GCF of 8. What is the LCM of x and y?

**Ans.** ____________________

**5 pts 3.** Let $g$ be a whole number such that $g-1$ and $g+1$ are prime numbers. $(g-1)+(g+1)$ is equal to $f-1$ where $f$ is a prime number. Find the largest possible value of $f-g$, given that $g<30$.

**Ans.** ____________________

### Number Theory

**1.** $1(216)+2(36)+3(6)+4=216+72+18+4=220+90=310.$ **Ans. 310**

**2.** $768=8\cdot(8\cdot 12), LCM=96; (8\cdot 2)(8\cdot 6), LCM=48$, but GCF = 16; $(8\cdot 3)(8\cdot 4), LCM=96.$

So the LCM = 96. **Ans. 96**

**3.** Prime numbers less than 30 are 2, 3, 5, 7, 11, 13, 17, 19, 23, 29. 17 and 19 fit $g+1$ and $g-1$, they add to 36. If $f-1=36$, then $f=37$ which is a prime. $f-g=37-18=19$. **Ans. 19**

## 4 Number Theory Oct 2018 (No Calculators)

**3 pts 1.** Find the sum of the prime factors of 3003.

**Ans.** ______________

**4 pts 2.** For how many positive integer values of $A$ less than 100 is the fraction $\frac{A}{100}$ reducible?

**Ans.** ______________

**5 pts 3.** Euler's totient function states that the number of positive integers less than positive integer N and relatively prime to N is equal to $N(1 - \frac{1}{p_1})(1 - \frac{1}{p_2})\ldots(1 - \frac{1}{p_k})$, where each of the $p_i$'s is a prime factor of N. Thus there are $300(1 - \frac{1}{2})(1 - \frac{1}{3})(1 - \frac{1}{5}) = 300(\frac{1}{2}\cdot\frac{2}{3}\cdot\frac{4}{5}) = 80$ positive integers less than 300 that are relatively prime to 300. How many positive integers less than 22,176 are relatively prime to 22,176?

**Ans.** ______________

## Number Theory

**1.** $3003 = 3(1001) = 3(11)(91) = 3(11)(7)(13)$. Their sum is 34. **Ans. 34**

**2.** It is reducible if A is a multiple of 2, 2 through 98, there are 49, or if A is an odd multiple of 5, from 5 t0 95 there are 10. 49 + 10 = 59. **Ans. 59**

**3.** $22{,}176 = 9(2464) = 9(8)(308) = 9(8)(4)(77) = 2^5 \cdot 3^2 \cdot 7 \cdot 11$. Primes factors are 2, 3, 7, 11.

$22{,}176\left(\frac{1}{2}\right)\left(\frac{2}{3}\right)\left(\frac{6}{7}\right)\left(\frac{10}{11}\right) = 5760$, using a calculator. **Ans. 5760**

## 4 Number Theory Oct 2017 (No Calculators)

**3 pts 1.** Convert $4321_5$ to a base 10 numeral.

**Ans.** ______________

**4 pts 2.** What is the smallest positive number which has 8 factors?

**Ans.** ______________

**5 pts 3.** Find the mixed number in simplest form which is the sum of $\frac{249}{332}$ and $\frac{469}{603}$.

**Ans.** ______________

## Number Theory

1. $4321_5 = 4(125) + 3(25) + 2(5) + 1 = 500 + 75 + 10 + 1 = 586.$ **Ans. 586**

2. Possibilities are $n^7, m^3n^1, mnp$. To be smallest: n = 2, thus $2^7 = 128$. For $m^3n^1$, smallest when $2^3 3^1 = 8(3) = 24$. For *mnp*, $2(3)(5) = 30$. Smallest is 24. **Ans. 24**

3. Reducing the fractions: $\frac{249}{332}$: $332 - 249 = 83$. $249/83 = 3$. So this fraction is 3/4. For $\frac{469}{603}$ $603 - 469 = 134 = 2(67)$. $469/67 = 7$. So the fraction is 7/9. $\frac{3}{4} + \frac{7}{9} = \frac{27+28}{36} = \frac{55}{36}$. **Ans.** $1\frac{19}{36}$

## 4 Number Theory Oct 2016-2017

**3 pts 1.** How many positive integral factors does 504 have?

**Ans.** ____________________

**4 pts 2.** What is the unit's digit of the number resulting from the expansion of $(2017)^{2016}$?

**Ans.** ____________________

**5 pts 3.** Of the distinct two-digit prime numbers ending in 7, find the pair whose sum has the greatest number of factors.

**Ans.** ____________________

## Number Theory

**1.** $504 = 3\cdot3\cdot2\cdot2\cdot2\cdot7 = 3^2 2^3 7^1$, so # factors $= 3\cdot4\cdot2 = 24$. **Ans. 24**

**2.** The pattern of unit's digits is 7, 9, 3. 1, 7, 9, 3, 1. 2016/4 comes out even, so it is the 4th number, (1), in the pattern. **Ans. 1**

**3.** Making a cross product:

| | 17 | 37 | 47 | 67 | 97 |
|---|---|---|---|---|---|
| 17 | | | | | |
| 37 | 54 | | | | |
| 47 | 64 | 84 | | | |
| 67 | 84 | 104 | 114 | | |
| 97 | 114 | 134 | 144 | 164 | |

Of these sums, 144 has the most factors 5*3 = 15.

**Ans. 47 and 97**

## 4 Number Theory Oct 2015 (No Calculators)

**3 pts 1.** There is a positive prime number that is a factor of the sum of any three consecutive positive integers. What is the factor?

**Ans.** ____________________

**4 pts 2.** Find the sum of the positive integers less than 100 with an odd number of factors.

**Ans.** ____________________

**5 pts 3.** n and n + 61 are both perfect squares of consecutive natural numbers. m and m + 61 are both perfect cubes of consecutive natural numbers. What is the value of n – m?

**Ans.** ____________________

### Number Theory

1. If x is the first integer, then x + 1 and x + 2 are the next two. Adding makes 3x + 3, which makes 3(x + 1). 3 is the factor. **Ans. 3**

2. Any number of the form n2 has 3 factors, thus the perfect squares 1 + 4 + 9 + 16 + 25 + 36 + 49 + 64 + 81 = 285. n4 or n2m2 also have an odd number of factors. But these that are less than 100 are already in the sum. **Ans. 285**

3. Let x = the smaller of the consecutive integers that make n and n + 61. Then $(x + 1)^2 - x^2 = 61 \rightarrow x^2 + 2x + 1 - x^2 = 61 \rightarrow 2x = 60$, so x = 30 and n = 900. Let y be the smaller of the consecutive integers that make m and m + 61. Then $(m + 1)^3 - m^3 = 61 \rightarrow$

$m^3 + 3m^2 + 3m + 1 - m^3 = 61 \rightarrow m^2 + m - 20 = 0 \rightarrow (m - 4)(m + 5) = 0$. So m = 4 and m = 64.

n – m = 900 – 64 = 836. **Ans. 836**

## 4 Number Theory Oct 2014 (No Calculators)

**3 pts 1.** What is the greatest single-digit whole number that is a factor of the big number

123,456,789,987,654,321?

**Ans.** ______________

**4 pts 2.** Find the number of positive integer factors of the least common multiple of the set $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15\}$

**Ans.** ______________

**5 pts 3.** How many distinct ordered triples of integers (a, b, c) exist such that abc = 120?

**Ans.** ______________

**Number Theory**

1. The sum of the digits is 90. So the greatest single greatest factor is 9. **Ans. 9**

2. The LCM is $2^3 \cdot 3^2 \cdot 5 \cdot 7 \cdot 11 \cdot 13$, so the number of positive integer factors is $4 \cdot 3 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 192$. **Ans. 192**

3. $120 = 2^3 \cdot 3 \cdot 5$. Since abc must equal 120, the number of ways to do it with positive integers is the number of ways to assign the three 2's, ${}_5C_2$, times the number of ways of assigning the 3, three, times the number of ways of assigning the 5, three: $10 \cdot 3 \cdot 3 = 90$.. Additionally there are 3 ways to assign the negative signs. So 90 + 3(90) = 360. **Ans. 360**

## 4 Number Theory Oct 2013 (No Calculators)

**3 pts 1.** Find the sum of the prime factors of 2184. Each distinct factor may be used on ly once in the sum.

**Ans.** ____________

**4 pts 2.** Find the value of the sum $x + y + z$, if

1. x, y, and z are relatively prime
2. x and y are not relatively prime
3. x and z are not relatively prime
4. y and z are not relatively prime
5. x, y and z are the smallest such natural number triplet.

**Ans.** ____________

**5 pts 3.** Find the x for the following base equation: $212_x + 264_{2x} = 166_{3x}$.

**Ans.** ____________

**Number Theory**

1. $2184 = 21(104) = 3(7)(4)(26) = 3(7)(4)(2)(13)$. $2 + 3 + 7 + 13 = 25$. **Ans. 25**

2. Using the smallest prime factors: 2(3), 2(5), 3(5): $6 + 10 + 15 = 31$. **Ans. 31**

3. $2x^2 + x + 2 + 8x^2 + 12x + 4 = 9x^2 + 18x + 6 \Rightarrow x^2 - 5x = 0$. So $x = 5$. **Ans. 5**