1. Determine convergence or divergence using any test

a. $\sum_{n=1}^{\infty} \frac{n^3}{n!}$

$\lim_{n\to\infty} \left| \frac{(n+1)^3}{(n+1)!} \cdot \frac{n!}{n^3} \right| = \lim_{n\to\infty} \frac{(n+1)^3}{n^3(n+1)} = \lim_{n\to\infty} \frac{(n+1)^2}{n^3}$

$= 0 < 1$ ✓

$\boxed{\text{converges}}$ by Ratio Test

b. $\sum_{n=1}^{\infty} \frac{n}{2n+1}$

$\lim_{n\to\infty} \frac{n}{2n+1} = \frac{1}{2} \neq 0$

$\boxed{\text{Diverges}}$ by nth term test

c. $\sum_{n=1}^{\infty} 2^{1/n}$

$\lim_{n\to\infty} 2^{1/n} = 1$

$\boxed{\text{Diverges}}$ by nth term test

d. $\sum_{n=1}^{\infty} \frac{\sin n}{n^2}$

$\frac{\sin n}{n^2} \leq \frac{1}{n^2}$ since $\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges by p-series

$\sum_{n=1}^{\infty} \frac{\sin n}{n^2}$ $\boxed{\text{converges}}$ by direct comparison test

e. $\sum_{n=1}^{\infty} \frac{1}{n+\sqrt{n}}$

$\sum_{n=1}^{\infty} \frac{1}{n+\sqrt{n}}$ compare w/ $\sum_{n=1}^{\infty} \frac{1}{n}$ which diverges by p-series

$\lim_{n\to\infty} \frac{\frac{1}{n+\sqrt{n}}}{\frac{1}{n}} = \lim_{n\to\infty} \frac{n}{n+\sqrt{n}} = 1$ since 1 is positive and finite

$\sum_{n=1}^{\infty} \frac{1}{n+\sqrt{n}}$ $\boxed{\text{diverges}}$

f. $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^2+1}}$ compare w/ $\sum \frac{1}{\sqrt{n^2}} = \sum \frac{1}{n}$ which diverges by p-series

$$\lim_{n\to\infty} \frac{\frac{1}{\sqrt{n^2+1}}}{\frac{1}{\sqrt{n}}} = \lim_{n\to\infty} \sqrt{\frac{n}{n^2+1}} = \sqrt{\lim_{n\to\infty} \frac{n}{n^2+1}} = 1$$ since 1 is finite & positive

$\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^2+1}}$ $\boxed{\text{diverges}}$

g. $\sum_{n=1}^{\infty} \frac{(-1)^n n}{\sqrt{n^2+1}}$

$$\lim_{n\to\infty} \frac{n}{\sqrt{n^2+1}} = 1$$ alternating series does not apply

$\boxed{\text{diverges}}$ by nth term test

b. $\sum_{n=1}^{\infty} \frac{2^n+4^n}{7^n}$ (Hint: $\sum a_n + b_n = \sum a_n + \sum b_n$)

$$\sum_{n=1}^{\infty} \frac{2^n+4^n}{7^n} = \sum_{n=1}^{\infty} \frac{2^n}{7^n} + \sum_{n=1}^{\infty} \frac{4^n}{7^n}$$

$$= \sum_{n=1}^{\infty} (2/7)^n + \sum_{n=1}^{\infty} (4/7)^n$$

$r = 2/7$ $r = 4/7$

$\boxed{\text{converges}}$ by geo