eas to take antiderivative of

Why Partial Fractions:

$\rightarrow \frac{3}{x-4} - \frac{2}{x+2} = \frac{3(x+2) - 2(x+4)}{(x+2)(x+4)} = \frac{x-2}{(x+2)(x+4)}$

$= \frac{3x + 6 - 2x - 8}{(x+2)(x+4)}$

* allows to go backwards

The degree of the numerator must be SMALLER than the degree of the denominator

Table 7.2

S.No.	Form of the rational function	Form of the partial fraction
1.	$\frac{px+q}{(x-a)(x-b)}, a \neq b$	$\frac{A}{x-a} + \frac{B}{x-b}$
2.	$\frac{px+q}{(x-a)^2}$	$\frac{A}{x-a} + \frac{B}{(x-a)^2}$
3.	$\frac{px^2+qx+r}{(x-a)(x-b)(x-c)}$	$\frac{A}{x-a} + \frac{B}{x-b} + \frac{C}{x-c}$
4.	$\frac{px^2+qx+r}{(x-a)^2(x-b)}$	$\frac{A}{x-a} + \frac{B}{(x-a)^2} + \frac{C}{x-b}$
5.	$\frac{px^2+qx+r}{(x-a)(x^2+bx+c)}$	$\frac{A}{x-a} + \frac{Bx+C}{x^2+bx+c}$,
	where $x^2 + bx + c$ cannot be factorised further	

Steps:

1. Set up with unknown constants; A, B, C,.. in numerator
2. Multiply by Denominator
3. Simplify
4. Group terms
5. create and solve system
6. plug A, B, C... in

1. Finding a Partial Fraction Decomposition:

a. $\frac{x+14}{(x-4)(x+2)} = \frac{A}{x-4} + \frac{B}{x+2}$

$x+14 = A(x+2) + B(x-4)$

① $x+14 = Ax + A2 + Bx - B4$

$x+14 = Ax + Bx + A2 - B4$

$1 = A + B$

$14 = 2A - 4B$

$-2 = -2A - 2B$

$14 = 2A - 4B$

$12 = -6B$

$B = -2$

$A = 3$

OR ②

$x+14 = A(x+2) + B(x-4)$

$x = -2$

$12 = -6B$

$-2 = B$

$x = 4$

$18 = 6A$

$3 = A$

$$\boxed{\frac{3}{x-4} + \frac{-2}{x+2}}$$

b. $\frac{x-18}{x(x-3)^2}$

$= \frac{A}{x} + \frac{B}{x-3} + \frac{C}{(x-3)^2}$

$x - 18 = A(x-3)^2 + B(x-3)x + Cx$

* can solve using option ① but need to foil

$x = 0$

$-18 = A(-3)^2$

$-18 = 9A$

$-2 = A$

$x = 3$

$-15 = 3C$

$-5 = C$

$x-18 = -2(x-3)^2 + Bx(x-3) - 5x$

$x = 1$

$-17 = -2(4) - 2B - 5$

$-17 = -13 - 2B$

$-4 = -2B$

$2 = B$

$$\boxed{\frac{-2}{x} + \frac{2}{x-3} - \frac{5}{(x-3)^2}}$$

b. $\int \frac{2x+16}{x^2+x-6}dx = \int\left(\frac{A}{x+3} + \frac{B}{x-2}\right)dx$

$2x+16 = A(x-2) + B(x+3)$

$x=2$ $\quad$ $x=-3$

$20 = 5B$ $\quad$ $-6+16 = -5A$

$4 = B$ $\quad$ $10 = -5A$

$-2 = A$

$= \int\left(\frac{-2}{x+3} + \frac{4}{x-2}\right)dx$

$= -2\ln|x+3| + 4\ln|x-2| + C$

$= 4\ln|x-2| - 2\ln|x+3| + C$

$= \ln(|x-2|)^4 - \ln(|x+3|)^2 + C$

$\boxed{= \ln\frac{(x-2)^4}{(x+3)^2} + C}$

c. $\int \frac{3}{x^2+9}dx$

★ partial fractions ★
won't help

$= 3\int \frac{1}{9\left(x^2/9 + 1\right)}dx$

$= \int \frac{1}{3}\frac{1}{\left(\frac{x}{3}\right)^2+1}dx$

$u = \frac{x}{3}$

$du = \frac{1}{3}dx$

$= \int \frac{1}{u^2+1}du$

$= \tan^{-1}u + C$

$\boxed{= \tan^{-1}(x/3) + C}$

c. $\frac{3x^2+17x+14}{(x-2)(x^2+2x+4)} = \frac{A}{x-2} + \frac{Bx+C}{x^2+2x+4}$

$3x^2+17x+14 = \frac{A}{x-2}(x-2)(x^2+2x+4) + \frac{Bx+C}{x^2+2x+4}(x-2)(x^2+2x+4)$

$3x^2+17x+14 = A(x^2+2x+4) + (Bx+C)(x-2)$

$x = 2$
$12 + 34 + 14 = 12A$
$60 = 12A$
$5 = A$

$x = 0$
$14 = 20 - 2C$
$-6 = -2C$
$3 = C$

$3x^2+17x+14 = 5(x^2+2x+4) + (Bx+3)(x-2)$
$x = 1$
$34 = 35 + (B+3)(-1)$
$-1 = -3 - B$
$-2 = B$

$$\boxed{= \frac{5}{x-2} + \frac{-2x+3}{x^2+2x+14}}$$

2. Antidifferentiating with Partial Fractions:

a. $\int \frac{3x^4+1}{x^2-1}dx$

* num degree larger

$$
\begin{array}{r|l}
 & 3x^2 + 3 \\
x^2-1 & 3x^4 \qquad +1 \\
 & -(3x^4 - 3x^2) \\
 & 3x^2 + 1 \\
 & -(3x^2 - 3) \\
 & 4
\end{array}
$$

$\int \frac{3x^4+1}{x^2-1}dx = \int\left(3x^2+3+\frac{4}{x^2-1}\right)dx$

$= \int(3x^2+3)dx + \int \frac{4}{x^2-1}dx$

$= x^3 + 3x + \int\left(\frac{A}{x-1} + \frac{B}{x+1}\right)dx$

$\frac{4}{x^2-1} = \frac{A}{x-1} + \frac{B}{x+1}$

$4 = A(x+1) + B(x-1)$

$x = -1$
$4 = -2B$
$-2 = B$

$x = 1$
$4 = 2A$
$2 = A$

$= x^3 + 3x + \int\left(\frac{2}{x-1} + \frac{-2}{x+1}\right)dx$

$= x^3 + 3x + 2\ln|x-1| - 2\ln|x+1| + C$

$$\boxed{= x^3 + 3x + 2\ln\left|\frac{x-1}{x+1}\right| + C}$$

4. The growth rate of a population P of bears in a newly established wildlife preserve is modeled by the differential equation $\frac{dP}{dt} = 0.008P(100 - P)$, where t is measured in years.
   a. What is the carrying capacity for bears in this wildlife preserve?

   100 bears

   b. What is the bear population when the population is growing the fastest?

   half the carrying capacity or 50 bears

   prove: $\frac{dP}{dt} = 0.8P - 0.008P^2$ $0 = 0.8 - 0.016P$

   $\frac{dP^2}{dt^2} = 0.8 - 0.016P$ $50 = P$ ✓

   c. What is the rate of change of the population when it is growing the fastest?

   $\frac{dP}{dt} = 0.008(50)(100 - 50)$

   $= 20$

   The rate of change of the population when it is growing the fastest is 20 bears that year

5. In 1985 and 1987, the Michigan Department of Natural REsources airlifted 61 moose from Algonquin Park, Ontario, to Marquette County in the Upper Peninsula. It was originally hoped that the population P would reach carrying capacity in about 25 years with a growth rate of

$$\frac{dP}{dt} = 0.0003P(1000 - P).$$

   a. According to the model, what is the carrying capacity?

   1,000

   b. With a calculator, generate a slope field for the differential equation.

Menu

3. Graph Entry/Edit

7. Diff Eq

$(y1)' = 0.003 \cdot y1 \cdot (1000 - y1)$

Window

$[0, 25] \times [0, 1000]$

* can add initial condition

3. Find the integral without using the technique of partial fractions:

a. $\int \frac{4x-3}{2x^2-3x+1} dx$

$u = 2x^2 - 3x + 1$

$du = (4x - 3)\, dx$

$= \int \frac{1}{u}\, du$

$= \ln|u| + C$

$\boxed{= \ln|2x^2 - 3x + 1| + C}$

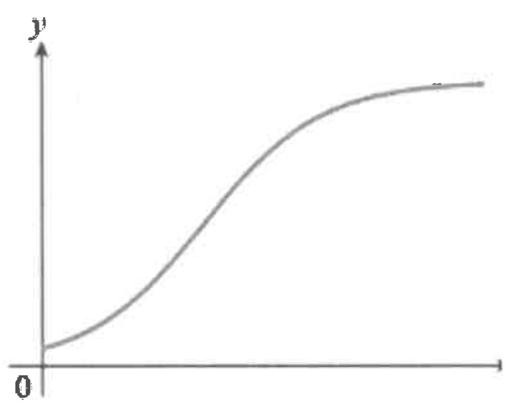
Integration Techniques

1) U-sub

2) By parts

3) Partial fractions

Logistic Curve



Can be modeled by $\frac{dP}{dt} = kp$ for some $k > 0$

If we want the growth rate to approach 0 as P approaches a maximal **carrying capacity** M, then:

$$\frac{dP}{dt} = kp(M - P)$$

The General Logistic Formula

The solution of the general logistic differential equation

$$\frac{dP}{dt} = kP(M - P)$$

is

$$P = \frac{M}{1 + Ae^{-(Mk)t}}$$

where A is a constant determined by an appropriate initial condition. The **carrying capacity** M and the **growth constant** k are positive constants.

6. The growth of the population of Aurora, CO, for the years between 1950 and 2003 was roughly logistic, satisfying the differential equation $\frac{dP}{dt} = P(0.1 - 3.125 \times 10^{-7}P)$. Model the growth with a logistic function, using the initial condition $P(0) = 12,800$.

$$\frac{dP}{dt} = (3.125 \times 10^{-7})\,P\,(320,000 - P)$$

$$P = \frac{320,000}{1 + Ae^{-(320,000)(3.125 \times 10^{-7})t}}$$

$$P = \frac{320,000}{1 + 24e^{-0.1}}$$

$$12,800 = \frac{320,000}{1 + Ae^{0}}$$

$$1 + A = \frac{320,000}{12,800}$$

$A = 24$