No calculator, unless the question says "Calc" in top right corner.

1) For what value of x does the function $f(x)=(x-2)(x-3)^2$ have a relative maximum? 1993 AB 15 Calc

(A) -3 (B) $-\frac{7}{3}$ (C) $-\frac{5}{2}$ (D) $\frac{7}{3}$ (E) $\frac{5}{2}$

$f'(x) = (x-3)(3x-7)$

$x = 3, \ 7/3$

① 2nd derivative test

$f''(x) = 6x - 16$

$f''(3) > 0$ ∪ $f''(7/3) < 0$ ∩

OR

② first derivative test

$+ \quad - \quad +$ f'

$\frac{7}{3} \quad 3$

2) If g is a differentiable function such that $g(x)<0$ for all real numbers x and if $f'(x)=(x^2-4)g(x)$, which of the following is true? 1998 AB 89 Calc

(A) f has a relative maximum at $x=-2$ and a relative minimum at $x=2$.
(B) f has a relative minimum at $x=-2$ and a relative maximum at $x=2$.
(C) f has relative minima at $x=-2$ and at $x=2$.
(D) f has relative maxima at $x=-2$ and at $x=2$.
(E) It cannot be determined if f has any relative extrema.

$f'(x) = (x^2-4)\,g(x)$

$x = \pm 2$

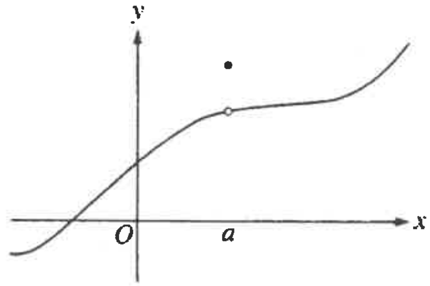
$(-)(-)$

$- \quad + \quad -$ f'

$-2 \quad 2$

min max

1998 AB 76
Calc



3) The graph of a function f is shown above. Which of the following statements about f is false?

(A) f is continuous at $x = a$.

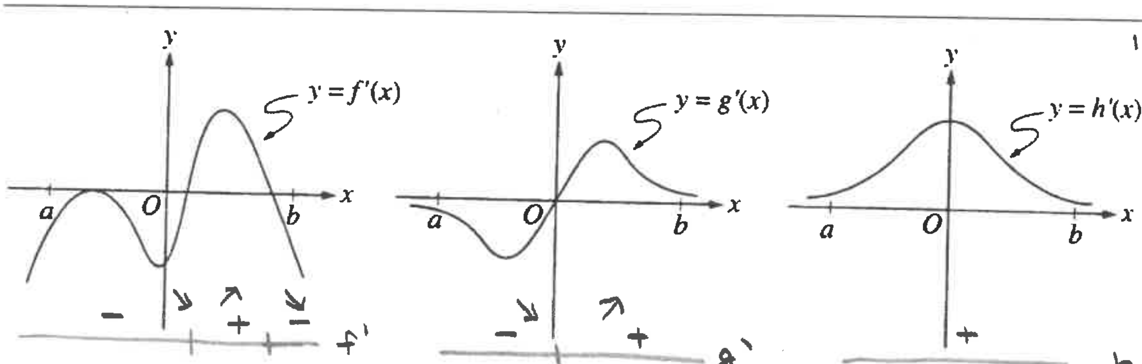
(B) f has a relative maximum at $x = a$.

(C) $x = a$ is in the domain of f.

(D) $\lim_{x \to a^+} f(x)$ is equal to $\lim_{x \to a^-} f(x)$.

(E) $\lim_{x \to a} f(x)$ exists.

1998 AB 79
Cak



4) The graphs of the derivatives of the functions f, g, and h are shown above. Which of the functions f, g, or h have a relative maximum on the open interval $a < x < b$?

(A) f only
(B) g only
(C) h only
(D) f and g only
(E) f, g, and h

5) A point moves on the x-axis in such a way that its velocity at time t $(t>0)$ is given by $v=\frac{\ln t}{t}$. 1969 AB 19

At what value of t does v attain its maximum?

(A) 1 (B) $e^{\frac{1}{2}}$ (C) e (D) $e^{\frac{3}{2}}$

(E) There is no maximum value for v.

$$v'(t)=\frac{(t)(1/t)-\ln t\,(1)}{t^2}$$

$$0=\frac{1-\ln t}{t^2}$$

$\ln t = 1$

$e^1 = t$

$t=0 \quad t=e$

und, $+$, $-$; v' ; 0, e

6) The absolute maximum value of $f(x)=x^3-3x^2+12$ on the closed interval $[-2,4]$ occurs at $x=$ 1988 AB 33

(A) 4 (B) 2 (C) 1 (D) 0 (E) -2

$f'(x) = 3x^2 - 6x$

$0 = 3x(x-2)$

$x = 0, 2$

$+$, $-$, $+$; f' ; 0, 2

$f(-2) = -8$ $\quad f(2) = 8$

$f(0) = 12$ $\quad f(4) = 28$

7) The *derivative* of $f(x)=\frac{x^4}{3}-\frac{x^5}{5}$ attains its maximum value at $x=$ 1973 AB 10

(A) -1 (B) 0 (C) 1 (D) $\frac{4}{3}$ (E) $\frac{5}{3}$

$f'(x) = \frac{4}{3}x^3 - x^4$ max of

$f''(x) = 4x^2 - 4x^3$

$0 = 4x^2(1-x)$

$x = 0, 1$

$+$, $+$, $-$; 0, 1

8) For what value of k will $x+\frac{k}{x}$ have a relative maximum at $x=-2$? 1969 AB 7

(A) -4 (B) -2 (C) 2 (D) 4 (E) None of these

$0 = f'(-2)$

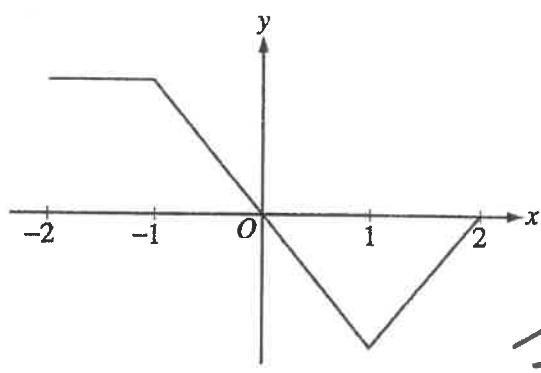
$f'(x) = 1 - kx^{-2}$

$f'(-2) = 1 - \frac{k}{4}$

$0 = 1 - \frac{k}{4}$

$k = 4$

9) 2003 AB 7



Graph of f'

$+$ $-$ f'

0 2

The graph of f', the derivative of the function f, is shown above. Which of the following statements is true about f ?

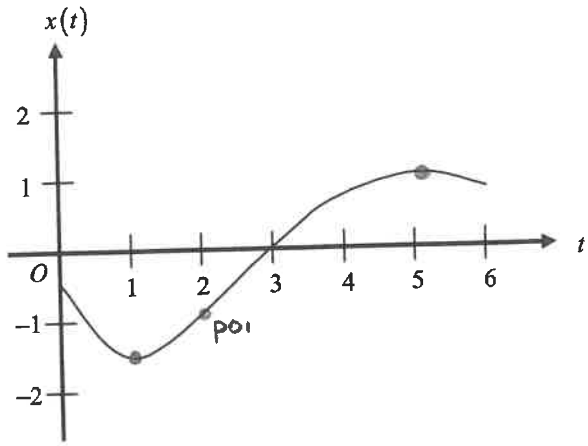
(A) f is decreasing for $-1 \le x \le 1$.

(B) f is increasing for $-2 \le x \le 0$.

(C) f is increasing for $1 \le x \le 2$.

(D) f has a local minimum at $x = 0$.

(E) f is not differentiable at $x = -1$ and $x = 1$.



2008 AB 21

10) A particle moves along a straight line. The graph of the particle's position $x(t)$ at time t is shown above for $0<t<6$. The graph has horizontal tangents at $t=1$ and $t=5$ and a point of inflection at $t=2$. For what values of t is the velocity of the particle increasing?

slope

$f'' > 0$

so concave up

(A) $0<t<2$

(B) $1<t<5$

(C) $2<t<6$

(D) $3<t<5$ only

(E) $1<t<2$ and $5<t<6$

$x = \pm 2$

1997 AB 13

11) Let f be a function defined for all real numbers x. If $f'(x)=\dfrac{|4-x^2|}{x-2}$, then f is decreasing on the interval

(A) $(-\infty,2)$ (B) $(-\infty,\infty)$ (C) $(-2,4)$ (D) $(-2,\infty)$ (E) $(2,\infty)$

$-$ $-$ $+$ f'

-2 2

12) The function f given by $f(x) = x^3 + 12x - 24$ is 1993 AB 27 Calc

(A) increasing for $x < -2$, decreasing for $-2 < x < 2$, increasing for $x > 2$
(B) decreasing for $x < 0$, increasing for $x > 0$
(C) increasing for all x
(D) decreasing for all x
(E) decreasing for $x < -2$, increasing for $-2 < x < 2$, decreasing for $x > 2$

$f'(x) = 3x^2 + 12$

$0 = 3(x^2 + 4)$

no cp

$\overset{+}{\rule{3cm}{0.4pt}}$ f'

13) At $x = 0$, which of the following is true of the function f defined by $f(x) = x^2 + e^{-2x}$? 1969 AB 21

(A) f is increasing.
(B) f is decreasing.
(C) f is discontinuous.
(D) f has a relative minimum.
(E) f has a relative maximum.

$f'(x) = 2x - 2e^{-2x}$

$f'(0) = 2(0) - 2e^{0}$

$= -2$

14) What is the x-coordinate of the point of inflection on the graph of $y = \frac{1}{3}x^3 + 5x^2 + 24$? 1998 AB 1

(A) 5 (B) 0 (C) $-\frac{10}{3}$ (D) -5 (E) -10

$y' = x^2 + 10x$

$y'' = 2x + 10$

$0 = 2x + 10$

$x = -5$

$\overset{-}{\rule{1.2cm}{0.4pt}}\underset{-5}{|}\overset{+}{\rule{1.2cm}{0.4pt}}$ f''

15) The graph of the function $y = x^3 + 6x^2 + 7x - 2\cos x$ changes concavity at $x =$ 1997 AB 77 Calc

(A) -1.58 (B) -1.63 (C) -1.67 (D) -1.89 (E) -2.33

$y' = 3x^2 + 12x + 7 + 2\sin x$

$y'' = 6x + 12 + 2\cos x$

$0 = 6x + 12 + 2\cos x$

$x \approx -1.894$

16) If $f''(x) = x(x+1)(x-2)^2$, then the graph of f has inflection points when $x =$ 1998 AB 19

(A) -1 only (B) 2 only (C) -1 and 0 only (D) -1 and 2 only (E) -1, 0, and 2 only

$x = 0, -1, 2$

+ − + + f''

-1 0 2

17) The function f has first derivative given by $f'(x) = \dfrac{\sqrt{x}}{1 + x + x^3}$. What is the x-coordinate of the inflection point of the graph of f? 2003 AB 87 Calc

(A) 1.008 (B) 0.473 (C) 0 (D) -0.278 (E) The graph of f has no inflection point.

*all on calc

1) Take derivative

2) solve w/ ↑ set to zero

18) The derivative of the function f is given by $f'(x) = x^2 \cos(x^2)$. How many points of inflection does the graph of f have on the open interval $(-2, 2)$? 2008 AB 80 Calc

(A) One (B) Two (C) Three (D) Four (E) Five

* all on calc → see #17 for steps

* be aware of interval

and be careful of counting & scrolling on calculator

19) The graph of $y = \dfrac{-5}{x-2}$ is concave downward for all values of x such that 1988 AB 4

(A) $x < 0$ (B) $x < 2$ (C) $x < 5$ (D) $x > 0$ (E) $x > 2$

$y = -5(x-2)^{-1}$

$y' = 5(x-2)^{-2}$

$y'' = -10(x-2)^{-3}$

$0 = -10(x-2)^{-3}$

poi $x = 2$

y'': $+$ | 2 | $-$

20) The graph of $y = 3x^4 - 16x^3 + 24x^2 + 48$ is concave down for 1997 AB 5

(A) $x < 0$

(B) $x > 0$

(C) $x < -2$ or $x > -\dfrac{2}{3}$

(D) $x < \dfrac{2}{3}$ or $x > 2$

(E) $\dfrac{2}{3} < x < 2$

$y' = 12x^3 - 48x^2 + 48x$

$y'' = 36x^2 - 96x + 48$

$0 = 12(3x^2 - 8x + 4)$

$0 = (3x-2)(x-2)$

$x = 2/3,\ 2$

y'': $+$ | $\frac{2}{3}$ | $-$ | 2 | $+$

2003 AB 90 Calc

21) For all x in the closed interval $[2, 5]$, the function f has a positive first derivative and a negative second derivative. Which of the following could be a table of values for f? increasing concave down

(A)

x	$f(x)$	m
2	7	
3	9	2
4	12	3
5	16	4

cup

(B)

x	$f(x)$	m
2	7	
3	11	4
4	14	3
5	16	2

c down

(C)

x	$f(x)$
2	16
3	12
4	9
5	7

decreasing

(D)

x	$f(x)$
2	16
3	14
4	11
5	7

decreasing

(E)

x	$f(x)$
2	16
3	13
4	10
5	7

decreasing

---

22) . The function f is continuous on the closed interval $[2, 4]$ and twice differentiable on the open interval $(2, 4)$. If $f'(3) = 2$ and $f''(x) < 0$ on the open interval $(2,4)$, which of the following could be a table of values for f? 2008 AB 90 Calc

slope at 3 = 2

concave down

(A)

x	$f(x)$	m
2	2.5	
3	5	2.5
4	6.5	1.5

cdown

~~(B)~~

x	$f(x)$	
2	2.5	
3	5	2.5
4	7	2

cdown

slope cant be 2 at x = 3

~~(C)~~

x	$f(x)$	
2	3	
3	5	2
4	6.5	1.5

cdown

slope cant be 2 at x = 3

~~(D)~~

x	$f(x)$	
2	3	
3	5	2
4	7	2

linear

~~(E)~~

x	$f(x)$	
2	3.5	
3	5	1.5
4	7.5	2.5

cup

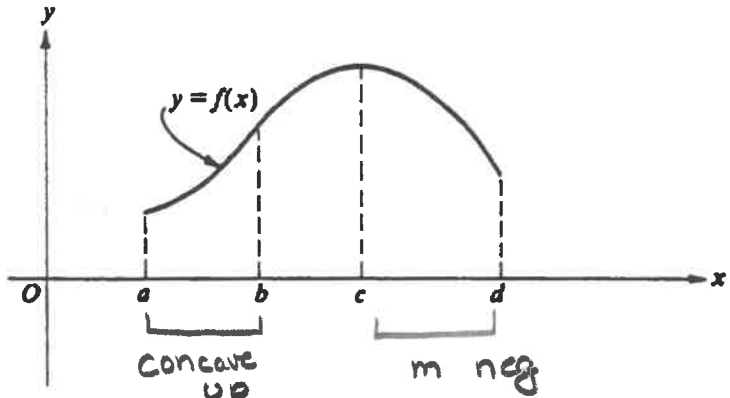
* can also reason w/ MVT

$$f'(c) \quad \frac{f(4) - f(2)}{4 - 2} = 2$$

but $f'(3) = 2$

and f' decreasing

23) 1988 AB 8



The graph of $y = f(x)$ is shown in the figure above. On which of the following intervals are

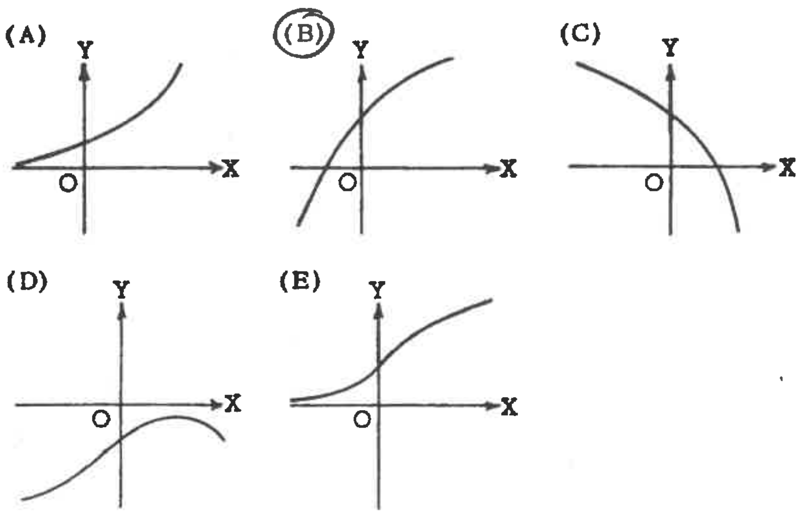
m pos/ increasing function $\frac{dy}{dx} > 0$ and $\frac{d^2y}{dx^2} < 0$? concave down

I. $a < x < b$
II. $b < x < c$
III. $c < x < d$

(A) I only (B) II only (C) III only (D) I and II (E) II and III

24) If y is a function of x such that $y' > 0$ (increasing) for all x and $y'' < 0$ (concave down) for all x, which of the following could be part of the graph of $y = f(x)$?

1969 AB 16



Calc
2003 AB 80

2. The function f is continuous for $-2 \le x \le 1$ and differentiable for $-2 < x < 1$. If $f(-2) = -5$ and $f(1) = 4$, which of the following statements could be false?

(A) There exists c, where $-2 < c < 1$, such that $f(c) = 0$. continuous function

(B) There exists c, where $-2 < c < 1$, such that $f'(c) = 0$. not guaranteed to have max/min/ critical point

(C) There exists c, where $-2 < c < 1$, such that $f(c) = 3$. continuous function

(D) There exists c, where $-2 < c < 1$, such that $f'(c) = 3$. ✓

(E) There exists c, where $-2 \le c \le 1$, such that $f(c) \ge f(x)$ for all x on the closed interval $-2 \le x \le 1$. continuous so must have absolute max on $[-2, 1]$ by EVT

* MVT

$$f'(c) = \frac{f(1) - f(-2)}{1 - (-2)} = \frac{4 - (-5)}{3} = \frac{9}{3} = 3$$

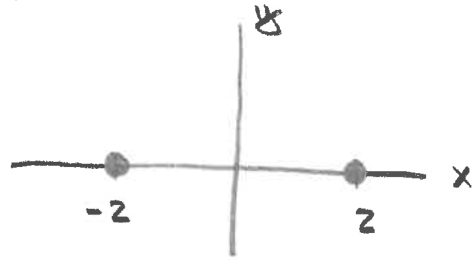
There exists a $c \in [-2, 1]$ such that $f'(c) = 3$

2008 AB 89 calc

26) The function f is continuous for $-2 \le x \le 2$ and $f(-2) = f(2) = 0$. If there is no c, where $-2 < c < 2$, for which $f'(c) = 0$, which of the following statements must be true?

(A) For $-2 < k < 2$, $f'(k) > 0$.

(B) For $-2 < k < 2$, $f'(k) < 0$.



(C) For $-2 < k < 2$, $f'(k)$ exists.

(D) For $-2 < k < 2$, $f'(k)$ exists, but f' is not continuous.

(E) For some k, where $-2 < k < 2$, $f'(k)$ does not exist. ✓

* could be a corner *

why we check for f' undefined

27) If $y = 2x - 8$, what is the minimum value of the product xy? 1997 AB 82 Calc

(A) -16 (B) -8 (C) -4 (D) 0 (E) 2

$M = x(2x-8)$

$= 2x^2 - 8x$

$M' = 4x - 8$

$0 = 4x - 8$ $M = -8$

$x = 2$ $y = -4$

28) The point on the curve $x^2 + 2y = 0$ that is nearest the point $\left(0, -\frac{1}{2}\right)$ occurs where y is 1969 AB 11

(A) $\frac{1}{2}$ (B) 0 (C) $-\frac{1}{2}$ (D) -1 (E) none of the above

$D = \sqrt{(x-0)^2 + (y + 1/2)^2}$ $y = -\frac{x^2}{2}$

$D = \sqrt{x^2 + (-x^2/2 + 1/2)^2}$

$M = x^2 + (-1/2\, x^2 + 1/2)^2$ $x = 0$

$y = 0$

$M' = 2x + 2(-1/2\, x^2 + 1/2)(-x)$

$0 = 2x + x^3 - x$ $0 = x + x^3 = x(1 + x^2)$

29) Which is the best of the following polynomial approximations to $\cos 2x$ near $x = 0$? 1969 AB 37

(A) $1 + \frac{x}{2}$ (B) $1 + x$ (C) $1 - \frac{x^2}{2}$ (D) $1 - 2x^2$ (E) $1 - 2x + x^2$

$f(x) = \cos 2x$ $f(0) = 1$ Ignore

$f'(x) = -2\sin 2x$ $y = 1 +$

$f'(0) = -2\sin 0$

$= 0$

2008 AB 24

The function f is twice differentiable with $f(2)=1$, $f'(2)=4$, and $f''(2)=3$. What is the value of the approximation of $f(1.9)$ using the line tangent to the graph of f at $x=2$?

(A) 0.4 (B) 0.6 (C) 0.7 (D) 1.3 (E) 1.4

$L(x) = f(2) + f'(2)(a-2)$

$= 1 + 4(1.9-1)$

$= 1 + 4(-0.1)$

$= 1 - 0.4$

$= 0.6$

31) The approximate value of $y=\sqrt{4+\sin x}$ at $x=0.12$, obtained from the tangent to the graph at $x=0$, is 1969 AB 36

(A) 2.00 (B) 2.03 (C) 2.06 (D) 2.12 (E) 2.24

$y' = \frac{1}{2}(4+\sin x)^{-1/2} \cos x$

$y(0) = \sqrt{4+\sin 0} = 2$

$y'(0) = \frac{\cos 0}{2\sqrt{4+\sin 0}} = \frac{1}{2\sqrt{4}} = \frac{1}{4}$

$$\begin{array}{r} 0.25 \\ 0.12 \\ \hline 0050 \\ 0250 \\ .0300 \end{array}$$

$L(x) = 2 + \frac{1}{4}(x-0)$

$= 2 + 1/4\, x$

$L(0.12) = 2 + 0.25(0.12)$

$= 2 + 0.03$

$= 2.03$

32) The radius r of a sphere is increasing at the uniform rate of 0.3 inches per second. At the instant when the surface area S becomes 100π square inches, what is the rate of increase, in cubic inches per second, in the volume V? $\left(S = 4\pi r^2 \text{ and } V = \frac{4}{3}\pi r^3\right)$ 1973 AB 26

(A) 10π (B) 12π (C) 22.5π (D) 25π (E) 30π

$\frac{dr}{dt} = 0.3$ in/sec

$SA = 100\pi$ in^2

$\frac{dV}{dt} = ?$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$\frac{dV}{dt} = 4\pi(5)^2(0.3)$

$= 100\pi \frac{3}{10}$

$= 30\pi$

$100\pi = 4\pi r^2$

$25 = r^2$

$5 = r$

1985 AB 31

33) The volume of a cone of radius r and height h is given by $V = \frac{1}{3}\pi r^2 h$. If the radius and the height both increase at a constant rate of $\frac{1}{2}$ centimeter per second, at what rate, in cubic centimeters per second, is the volume increasing when the height is 9 centimeters and the radius is 6 centimeters?

(A) $\frac{1}{2}\pi$ (B) 10π (C) 24π (D) 54π (E) 108π

$\frac{dr}{dt} = \frac{1}{2}$ cm/sec

$\frac{dh}{dt} = \frac{1}{2}$ cm/sec

$\frac{dV}{dt} = ?$ when $h = 9$ $r = 6$

$V = \frac{1}{3}\pi r^2 h$

$\frac{dV}{dt} = \frac{1}{3}\pi\left[2rh\frac{dr}{dt} + r^2\frac{dh}{dt}\right]$

$= \frac{1}{3}\pi\left(2(6)(9)(1/2) + 6^2(1/2)\right)$

$= \frac{1}{3}\pi(54 + 18) = \frac{72}{3}\pi$

34) The radius of a circle is increasing at a constant rate of 0.2 meters per second. What is the rate of increase in the area of the circle at the instant when the circumference of the circle is 20π meters?

2003 AB 78
Calc

(A) 0.04π m^2/sec

(B) 0.4π m^2/sec

(C) 4π m^2/sec

(D) 20π m^2/sec

(E) 100π m^2/sec

$\frac{dr}{dt} = 0.2$

$\frac{dA}{dt} = ?$

$C = 20\pi$

$20\pi = 2\pi r$

$10 = r$

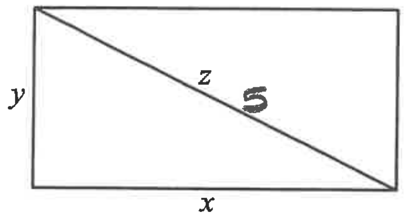
$A = \pi r^2$

$\frac{dA}{dt} = 2\pi r\frac{dr}{dt}$

$= 2\pi(10)(0.2)$

$= 4\pi$

1988 AB 40



35) The sides of the rectangle above increase in such a way that $\frac{dz}{dt} = 1$ and $\frac{dx}{dt} = 3\frac{dy}{dt}$. At the instant when $x = 4$ and $y = 3$, what is the value of $\frac{dx}{dt}$?

(A) $\frac{1}{3}$ (B) 1 (C) 2 (D) $\sqrt{5}$ (E) 5

$$x^2 + y^2 = z^2$$

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}$$

$$2(4)\frac{dx}{dt} + 2(3)\frac{1}{3}\frac{dx}{dt} = 2(5)(1)$$

$$8\frac{dx}{dt} + 2\frac{dx}{dt} = 10$$

$$10\frac{dx}{dt} = 10$$

$$\frac{dx}{dt} = 1$$