

6.1 Estimating with Finite Sums

- Suppose from the 2nd to 4th hour of your road trip you travel with cruise control set to exactly 70 mph for that two hour stretch. How far have you traveled during this time?

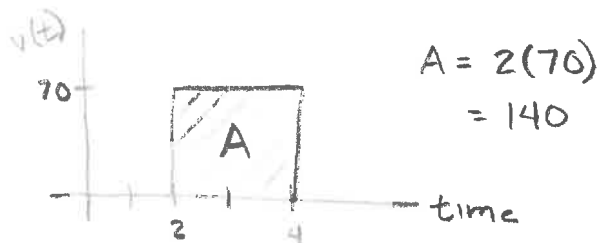
$$v = \frac{d}{t}$$

$$d = v(t)$$

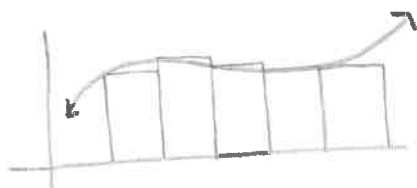
$$d = 70(2)$$

$$= 140 \text{ mi}$$

- Sketch a graph modeling the situation in the above example. Geometrically, how can we indicate the total distance traveled?



Now what happens if the distance is not constant?



Riemann Sums

- RRAM (Right Rectangular Approximation Method)
 - Use the right endpoint of each subinterval to determine the height of each rectangle.
- LRAM (Left Rectangular Approximation Method)
 - Use the left endpoint of each subinterval to determine the height of each rectangle.
- MRAM (Midpoint Rectangular Approximation Method)
 - Use the middle of each subinterval to determine the height of each rectangle.

*In each case the height of the rectangle is the **function value** at the selected x-value.

Steps:

- Divide (or partition) the interval into n subintervals
 - The partition does not have to be equal length, but will be for now
- Create n rectangles whose base equals the width of each subinterval and whose height is determined by the function value at the left endpoint, right endpoint, or midpoint of the subinterval.
- Find the area of n rectangles and add them together.

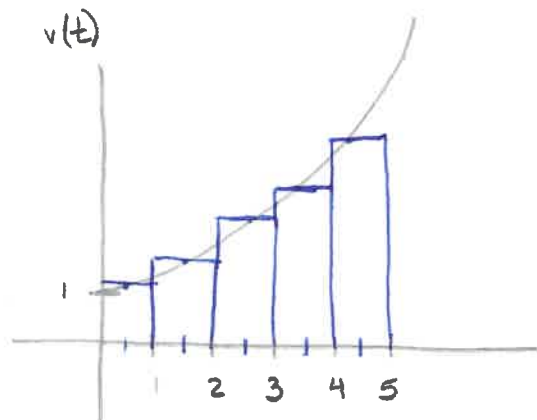
Insert Function into calc:

- 1) Menu
- 2) Actions
- 3) Define
- 4) $a(x) =$ function
- 5) enter

AB Calculus

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3. A particle starts at $x = 0$ and moves along the x -axis with velocity $v(t) = t^2 + 1$ for time $t \geq 0$. Where is the particle at $t = 5$? Approximate the area under the curve using five rectangles of equal width and heights determined by the midpoints of the intervals.



$$\text{partition} = \frac{5-0}{5} = 1$$

midpoints of partition

	$[0, 1]$	$[1, 2]$	$[2, 3]$	$[3, 4]$	$[4, 5]$
midpoint	$1/2$	$3/2$	$5/2$	$7/2$	$9/2$

$$A = (1 \cdot v(1/2)) + (1 \cdot v(3/2)) + (1 \cdot v(5/2)) + (1 \cdot v(7/2)) + (1 \cdot v(9/2))$$

$$A = \left[\left(\frac{1}{2} \right)^2 + 1 \right] + \left[\left(\frac{3}{2} \right)^2 + 1 \right] + \left[\left(\frac{5}{2} \right)^2 + 1 \right] + \left[\left(\frac{7}{2} \right)^2 + 1 \right] + \left[\left(\frac{9}{2} \right)^2 + 1 \right]$$

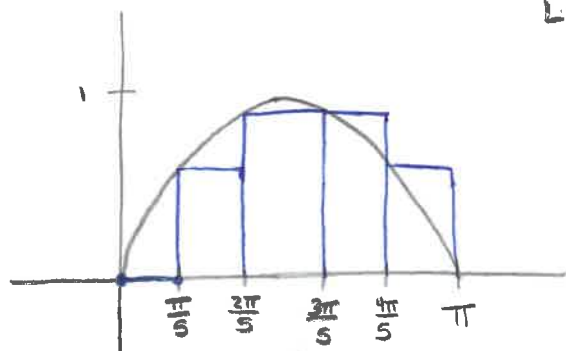
$$= \frac{5}{4} + \frac{13}{4} + \frac{29}{4} + \frac{53}{4} + \frac{85}{4}$$

$$= \frac{185}{4} = 46.25$$

* more accurate than a
LRAM or RRAM

4. Use LRAM with $n = 5$ to estimate the area of the region enclosed between the graph of f and the x -axis for $a \leq x \leq b$

$$f(x) = \sin x, a = 0, b = \pi$$



LRAM \rightarrow start
w/ left endpoint

$A = \text{width} \cdot \text{height}$
 \uparrow partition \uparrow function value

$$\begin{aligned} \text{partition} &= \frac{b-a}{n} \\ &= \frac{\pi-0}{5} \\ &= \pi/5 \end{aligned}$$

* 5 different areas *

$$A = \left(\frac{\pi}{5} \cdot f(0) \right) + \left(\frac{\pi}{5} \cdot f(\pi/5) \right) + \left(\frac{\pi}{5} \cdot f(2\pi/5) \right) + \left(\frac{\pi}{5} \cdot f(3\pi/5) \right) + \left(\frac{\pi}{5} \cdot f(4\pi/5) \right)$$

$$A = \left(\frac{\pi}{5} \cdot 0 \right) + \left(\frac{\pi}{5} \cdot \sin(\pi/5) \right) + \left(\frac{\pi}{5} \cdot \sin(2\pi/5) \right) + \left(\frac{\pi}{5} \cdot \sin(3\pi/5) \right) + \left(\frac{\pi}{5} \cdot \sin(4\pi/5) \right) \quad \leftarrow \text{doesn't go to } \pi$$

$$= 1.934$$

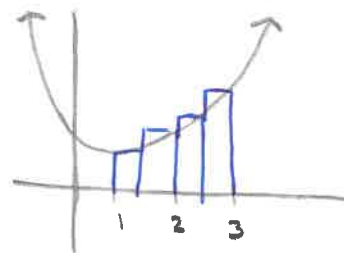
6.1 Estimating with Finite Sums

5. Use 4 rectangles to approximate the area under the graph of $y = x^2 - 2x + 2$ from $x = 1$ to $x = 3$. Use RRAM. start w/ right endpoint

$$\text{partition} = \frac{3-1}{4} = \frac{1}{2}$$

$$A = \left(\frac{1}{2} \cdot y(3) \right) + \left(\frac{1}{2} y\left(\frac{5}{2}\right) \right) + \left(\frac{1}{2} y(2) \right) + \left(\frac{1}{2} y\left(\frac{3}{2}\right) \right)$$

$$= 5.75$$



6. Use 4 rectangles to approximate the area under the graph of $y = x^2 - 2x + 2$ from $x = 1$ to $x = 3$. Use LRAM.

$$A = \frac{1}{2} \left(y(1) + y\left(\frac{3}{2}\right) + y(2) + y\left(\frac{5}{2}\right) \right)$$

$$= 3.75$$

7. Use 4 rectangles to approximate the area under the graph of $y = x^2 - 2x + 2$ from $x = 1$ to $x = 3$. Use MRAM.

	$\left[1, \frac{3}{2}\right]$	$\left[\frac{3}{2}, 2\right]$	$\left[2, \frac{5}{2}\right]$	$\left[\frac{5}{2}, 3\right]$
midpoint	$\frac{5}{4}$	$\frac{7}{4}$	$\frac{9}{4}$	$\frac{11}{4}$

$$A = \frac{1}{2} \left(y\left(\frac{5}{4}\right) + y\left(\frac{7}{4}\right) + y\left(\frac{9}{4}\right) + y\left(\frac{11}{4}\right) \right)$$

$$= 4.625$$

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8. The table below shows the velocity of a model train engine moving along a track for 10 sec. Estimate the distance traveled by the engine, using 10 subintervals of length 1 with (a) left-endpoint values (LRAM) and (b) right-endpoint values (RRAM)

partition: $\frac{10-0}{10} = 1$

Time (sec)	Velocity (in./sec)	Time (sec)	Velocity (in./sec)
0	0	6	11
1	12	7	6
2	22	8	2
3	10	9	6
4	5	10	0
5	13		

(a) $d = 1 \left[\overset{\text{left endpoint}}{0} + 12 + 22 + 10 + 5 + 13 + 11 + 6 + 2 + 6 \right]$

$d = 87 \text{ in}$

b) $d = 1 \left[12 + 22 + 10 + 5 + 13 + 11 + 6 + 2 + 6 + 0 \right]$
 \uparrow
 right endpoint

$= 87 \text{ in}$

9. You and a companion are driving along a twisty stretch of dirt road in a car whose speedometer works but whose odometer (mileage counter) is broken. To find out how long this particular stretch of road is, you record the car's velocity at 10-sec intervals, with the results shown in the table below. (The velocity was converted from mi/h to ft/sec using $30 \text{ mi/h} = 44 \text{ ft/sec}$.) Estimate the length of the road by averaging the LRAM and RRAM sums.

Time (sec)	Velocity (ft/sec)	Time (sec)	Velocity (ft/sec)
0	0	70	15
10	44	80	22
20	15	90	35
30	35	100	44
40	30	110	30
50	44	120	35
60	35		

LRAM:

$10(0 + 44 + 15 + 35 + 30 + 44 + 35 + 15 + 22 + 35 + 44 + 30)$
 $= 3490$

RRAM:

$10(44 + 15 + 35 + 30 + 44 + 35 + 15 + 22 + 35 + 44 + 30 + 35)$
 $= 3840$

Average = 3665 ft

AB Calculus
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Overestimate or Underestimate:

RRAM



LRAM



