

**Individuals Round 1 States 2017**

**3 pts 1.** Determine the Greatest Common Factor for 154 and 242.

**Ans.** \_\_\_\_\_

**4 pts 2.** From an ordinary deck of cards, cards with an alphabetical letter are removed. Find the probability that if a card was chosen at random from the remaining cards, that the card is an odd prime number.

**Ans.** \_\_\_\_\_

**5 pts 3.** Two lines intersect at (4, 5). Each line crosses the line  $x = 10$ . One crosses at (10, 10) and the other at (10, -3). Find the sum of the y-intercepts of both lines.

**Ans.** \_\_\_\_\_

**Individuals Round 2 States 2017**

**3 pts 1.** Solve  $ax + b = cx - 1$  for  $x$ .

**Ans.** \_\_\_\_\_

**4 pts 2.** Three equilateral triangles, each with a height of 6, form an isosceles trapezoid. Find the area of the trapezoid. Express in simplest form.

**Ans.** \_\_\_\_\_

**5 pts 3.** If the equation of the parabola, having points (6, 0), (2, -4) and (10, 0), takes on the form  $y = ax^2 + bx + c$  or  $x = ay^2 + by + c$ , find  $a + b + c$ .

**Ans.** \_\_\_\_\_

**Individuals Round 3 States 2017**

**3 pts 1.** The mean of the set of numbers  $x, 5, -2, x, 7, x, 9$  is 4. What is the sum of the mean, median, mode and range?

**Ans.** \_\_\_\_\_

**4 pts 2.** The volume of a rectangular pyramid is 40. The length and width of the base, and the height form three consecutive integers. Find the sum of these three dimensions.

**Ans.** \_\_\_\_\_

**5 pts 3.** Find all values of  $x$ , such that  $|x^2 - 4| \geq |4x|$ .

**Ans.** \_\_\_\_\_

**Individuals Round 4 States 2017**

**3 pts 1.** Find all pairs of positive prime numbers whose sum is 24. Express answer in ordered pair form: (smaller, larger)

**Ans.** \_\_\_\_\_

**4 pts 2.** The sum of the measures of angles  $A, B, C, D$  is  $180^\circ$ .  $\angle B$  is the complement of  $\angle C$ ,  $m\angle A = 2(m\angle B)$ ,  $m\angle C = 1.25(m\angle D)$ . Find the  $m\angle A + m\angle B$ .

**Ans.** \_\_\_\_\_

**5 pts 3.** Find all value(s) of  $x$  such that  $\sqrt{9x+12} - \sqrt{3x-2} = \sqrt{6x+2}$ .

**Ans.** \_\_\_\_\_

**Individuals Round 5 States 2017**

**3 pts 1.** Evaluate  $\begin{vmatrix} 2 & -3 & 4 \\ 4 & 2 & -3 \\ 3 & 4 & -2 \end{vmatrix}$ .

**Ans.** \_\_\_\_\_

**4 pts 2.** If  $x * y = 2x - y^2$  and  $x \# y = x/y$ , find  $(2 * 7) \# (7 * 2)$  in simplest form.

**Ans.** \_\_\_\_\_

**5 pts 3.** The ellipse  $9x^2 + 4y^2 - 18x + 8y - 23 = 0$  is moved 4 units to the right, then raised 5 units and finally rotated  $90^\circ$ . What are the coordinates of the endpoints of the major axis once it has been moved and rotated?

**Ans.** \_\_\_\_\_

**Individuals Round 6 States 2017**

**3 pts 1.** An item was sold for \$25.97 after a 6% sales tax was charged. How much of the cost of the item was for the sales tax?

**Ans.** \_\_\_\_\_

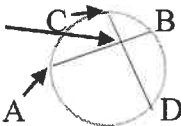
**4 pts 2.** If  $\frac{x^{a^2}}{x^{b^2}} = x^{24}$ , where  $x > 1$  and  $a + b = 4$ . Find the value of  $a - b$ .

**Ans.** \_\_\_\_\_

**5 pts 3.**  $\log_5 6 + \log_{625} 2 + \log_{25} 3 = \log_5 N$ . Find  $N$  in simplest form.

**Ans.** \_\_\_\_\_

**Team Round 1 States 2017**

**4 pts 1.** In the circle,  $PB = \frac{2}{3} CP$ . If  $AP = 4$ ,  (1) Ans. \_\_\_\_\_ **4 pts**  
find the measure of DP.

**4 pts 2.** A rectangular pyramid has base dimensions of 2 yards 1 foot 3 inches by 1 yard 2 inches. Its height is 16 inches. Find its volume in cubic feet. (2) Ans \_\_\_\_\_ **4 pts**

**6 pts 3.** The bill for Cable TV, Internet and telephone service is changing. Currently Cable is 60% of the bill. Internet service is 20% of the bill. Telephone service is 20% of the bill. Internet is increasing by 50% over its former price. Telephone service is staying at the same price. Cable is increasing by 25% of its former price. The bill for the first month of changes is \$250. What would have been the bill using the former charges? (3) Ans. \_\_\_\_\_ **6 pts**

**6 pts 4.** If  $x + y + 2z = 1$ ,  $3x - 4y - 5z = 4\frac{1}{2}$  and  $4x + 3y + 2z = 2$ , find  $x + y + z$ . (4) Ans. \_\_\_\_\_ **6 pts**

**6 pts 5.** Simplify:  $\frac{2x+3}{x+5} - \frac{3x+2}{x-5} + \frac{2x^2+13x+55}{x^2-25}$  (5) Ans. \_\_\_\_\_ **6 pts**

**8 pts 6.** At what point does the function  $f(x) = \frac{4}{x-1} - \frac{x+3}{x^2-1}$  cross one of its asymptotes? (6) Ans. \_\_\_\_\_ **8 pts**

**8 pts 7.** Series A =  $a_1, a_2, a_3, \dots$  ; Series B =  $b_1, b_2, b_3, \dots$  ; Series C =  $a_1 + b_1, a_2 + b_2, a_3 + b_3, \dots$  . In Series A: the 21<sup>st</sup> term is 345 and the 35<sup>th</sup> term is 527. In Series B: the 17<sup>th</sup> term is 287 and the 38<sup>th</sup> term is 602. What is the sum of the first 38 terms of C? (7) Ans. \_\_\_\_\_ **8 pts**

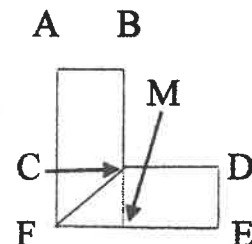
**8 pts 8.** Find all  $\theta$  where  $0^\circ \leq \theta < 360^\circ$  for which  $\cos^2 \theta \csc \theta - \cot \theta = \sin \theta$ . (8) Ans. \_\_\_\_\_ **8 pts**

**Team Round 2 States 2017**

**4 pts 1.**  $a^{5/4} \cdot b^{1/6} \cdot c^{3/2} = \sqrt[q]{a^n b^p c^r}$ , where n, p, r, q are the smallest possible integers. Find the value of n + p + r + q.

**(1) Ans.** \_\_\_\_\_ **4 pts**

**4 pts 2.** Given:  $\overline{AB} \parallel \overline{FE} \parallel \overline{CD}$ ,  $\overline{AF} \parallel \overline{BC} \parallel \overline{DE}$ ,  $\overline{BM} \perp \overline{FE}$ , BC = CD, DE = 4, M the midpoint of  $\overline{FE}$ , and FC = 5. Find the area of hexagon ABCDEF.



**(2) Ans.** \_\_\_\_\_ **4 pts**

**6 pts 3.** Ben takes 12 hrs. to do a certain job. Al can do the same job in 16 hrs. Al starts the job at 6:00 AM. Ben comes to work at 8:00 AM. Before they finish Al takes a 2 hour lunch break. To the nearest minute at what time will they finish the job? Specify AM or PM.

**(3) Ans.** \_\_\_\_\_ **6 pts**

**6 pts 4.** If  $\log_{x-2}(x^3 - 6x^2 + 12x - 8)(\sqrt{x-2}) = n$ , find n. **(4) Ans.** \_\_\_\_\_ **6 pts**

**6 pts 5.** If  $\frac{2+3i}{4-i}(1+i) + (2-i)^{-1} = a+bi$ , find a + b in simplest form.

**(5) Ans.** \_\_\_\_\_ **6 pts**

**8 pts 6.** Find the domain of  $f(x)$ , if  $f(x) = \frac{3}{1 - \frac{2}{x - \frac{1}{x-2}}}$ .

**(6) Ans.** \_\_\_\_\_ **8 pts**

**8 pts 7.** Quadrilateral ABCD has coordinates: A(-5, 12), B(-3, -9), C(10, -2) and D(16, 21).

Find the area of the quadrilateral.

**(7) Ans.** \_\_\_\_\_ **8 pts**

**8 pts 8.** If  $x \neq -3, -1, \text{ or } 3/5$ , find all value(s) of x so that  $\frac{x+2}{x+3} - \frac{x-2}{x+1} = \frac{2x+1}{5x-3}$ .

**(8) Ans.** \_\_\_\_\_ **8 pts**

**Blue Relay Seat A States 2017**

The lines  $8x - 5y = -3$  and  $4x - 3y = -5$  intersect at point A. Find the sum of the coordinates of A.

Pass back:  $2A$      $A =$  Your answer.

**Blue Relay Seat B States 2017**

Car A travels at 80 mph and car B travels at 90 mph on an oval 2 mile track. How many miles does B travel before he gains a lap on A?

Pass back:  $\frac{X+B}{2}$      $B =$  Your answer     $X =$  The number you receive.

**Blue Relay Seat C States 2017**

Two opposite angles of a kite measure  $60^\circ$  and  $90^\circ$ . If each of the longer pair of sides is 12, its area takes on the form  $m + p\sqrt{q}$ . Find the value of  $m + p + q$ .

Pass back:  $1.5X + C$      $C =$  your answer     $X =$  The number you receive.

**Blue Relay Seat D States 2017**

Find the largest value of  $x$  which does not satisfy:  $|2x - 5| < 4x + 3$ .

Pass back:  $DX - 2$      $D =$  Your answer     $X =$  The number you receive

**Blue Relay Seat E States 2017**

A sock drawer has two pairs of brown socks, 3 pairs of blue socks and 3 pairs of black socks. Two socks are selected at random from the drawer. What is the probability that they are both the same color?

Pass in:  $\frac{X}{E}$      $E =$  Your answer     $X =$  The number you receive

**Green Relay Seat A States 2017**

The lines  $9x + 7y = -3$  and  $4x + 5y = 10$  intersect at point B. Find the product of the coordinates of B.

Pass back:  $-\frac{1}{2}A$       A = Your answer

**Green Relay Seat B States 2017**

Car A travels at 80 mph and car B travels at 90 mph on an oval 2 mile track. How many miles does A travel before B gains a lap on A?

Pass back:  $2X - B$       B = Your answer      X = The number you receive.

**Green Relay Seat C States 2017**

Two opposite angles of a kite measure  $60^\circ$  and  $90^\circ$ . If each of the longer pair of sides is 6, its area takes on the form  $m + p\sqrt{q}$ . Find the value of  $m + p + q$ .

Pass back:  $2C - X$       C = Your answer      X = The number you receive

**Green Relay Seat D States 2017**

Find the smallest value of x which does not satisfy  $|2x - 5| > 4x + 3$ .

Pass back:  $X - 7D$       D = Your answer.      X = The number you receive.

**Green Relay Seat E States 2017**

A sock drawer has two pairs of brown socks, 3 pairs of blue socks and 3 pairs of black socks. Two socks are selected at random from the drawer. What is the probability that they are not the same color?

Pass in:  $\frac{X}{E}$       E = Your answer      X = The number you receive.

**Pink Relay Seat A States 2017**

If  $\frac{15x^3y^2 \cdot 16x^2y^5}{24x^4y^4} = ax^m y^n$ , find the value of  $amn$ .

Pass back:  $\frac{1}{3}A$       A = Your answer

**Pink Relay Seat B States 2017**

Mark has \$3.32. Larry has \$4.48. How much money should Mark give Larry, so that Mark will then have only one-half as much as Larry?

Pass back: 10BX      B = Your answer      X = The number you receive.

**Pink Relay Seat C States 2017**

An 8-15-17 triangle is similar to a smaller triangle whose shortest side is 6. What is the area of the smaller triangle?

Pass back: 4C - X      C = Your answer      X = The number you receive

**Pink Relay Seat D States 2017**

If  $\begin{bmatrix} 4 & 3 \\ 2 & -1 \\ 5 & 4 \end{bmatrix} \cdot \begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \\ e & f \end{bmatrix}$ , find the value of  $a + b + c + d + e + f$ .

Pass back:  $X - \frac{D}{3}$       D = Your answer      X = The number you receive.

**Pink Relay Seat E States 2017**

If  $0^\circ \leq \theta < 360^\circ$ , find the positive difference between the maximum and minimum values of the solutions for the equation  $\tan^2 \theta + \sin^2 \theta = 2 - \cos^2 \theta$ .

Pass in:  $X + \frac{E}{10}$       E = Your answer      X = The number you receive



**Yellow Relay Seat A States 2017**

If  $\frac{30x^5y^7 \cdot 24x^2y^4}{36x^3y^3} = ax^m y^n$ , find  $\frac{am}{n}$ .

Pass back: 2A      A = Your answer

**Yellow Relay Seat B States 2017**

Mark has \$3.50. Larry has \$4.30. How much money should Mark give Larry so that Mark will have one fifth as much as Larry?

Pass back: BX      B = Your answer      X = The number you receive.

**Yellow Relay Seat C States 2017**

A 9-12-21 triangle is similar to a larger triangle whose second largest side is 30. What is the perimeter of the larger triangle?

Pass back:  $\frac{4C+20}{X}$       C = Your answer      X = The number you receive.

**Yellow Relay Seat D States 2017**

If  $\begin{bmatrix} 4 & 3 \\ 2 & -1 \\ 5 & 4 \end{bmatrix} \cdot \begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \\ e & f \end{bmatrix}$ , find the value of  $a + b + c - (d + e + f)$ .

Pass back: 2X - D      D = Your answer      X = The number you receive.

**Yellow Relay Seat E States 2017**

If  $0^\circ \leq \theta < 360^\circ$ , find the positive difference between the maximum and minimum values for the solutions of the equation:  $\tan^2 \theta + \cos^2 \theta = 4 - \sin^2 \theta$ .

Pass in:  $X - \frac{E}{10}$       E = Your answer      X = The number you receive.

### Solutions – Individuals Round 1

1.  $154 = 2 \cdot 7 \cdot 11$ ,  $242 = 2 \cdot 11 \cdot 11$ . GCF =  $2(11) = 22$ . **Ans. 22**
2. Cards left, 2 through 10. 3, 5, 7 odd primes.  $12/36 = 1/3$ . **Ans. 1/3**
3. (4, 5), (10, 10)  $\rightarrow$  Slope =  $5/6$ ,  $y = 5/6x \rightarrow 5x - 6y = -10$ ,  $y\text{-i} = 5/3$ .  
 (4, 5), 10, -3)  $\rightarrow$  Slope =  $-8/6$ ,  $y = -8/6x \rightarrow 8x + 6y = 62$ ,  $y\text{-i} = 31/3$ . Sum =  $36/3$ . **Ans. 12**

### Individuals Round 2

1.  $ax + b = cx - 1 \rightarrow ax - cx = -b - 1 \rightarrow (a - c)x = -b - 1 \rightarrow x = \frac{-b-1}{a-c} = \frac{b+1}{c-a}$ . **Ans.  $\frac{b+1}{c-a}$**
2. Upper base is  $8\sqrt{3}$  long, lower base is  $4\sqrt{3}$  long. Area =  $1/2 (6)(12\sqrt{3}) = 36\sqrt{3}$ . **Ans.  $36\sqrt{3}$**
3. For zeroes:  $y = a(x - 6)(x - 10)$ , for (2, -4):  $-4 = a(-4)(-8)$ ,  $a = -1/8$ ,  $y = -1/8(x - 6)(x - 10)$   
 $y = -\frac{1}{8}x^2 + 2x - 7\frac{1}{2}$ .  $-\frac{1}{8} + 2 - 7\frac{1}{2} = 2 - 7 = 2 - 7\frac{5}{8} = -5\frac{5}{8}$  **Ans.  $-5\frac{5}{8}$**

### Individuals Round 3

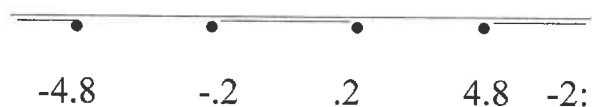
1.  $3x + 19 = 4(7)$ ,  $3x = 9$ , so  $x = 3$ . Mean = 4, median = 3, mode = 3, range = 11. **Ans. 21**
2.  $(x - 1)(x)(x+1)/3 = 40 \rightarrow x^3 - x = 120 \rightarrow x^3 - x - 120 = 0 \rightarrow (x - 4)(x - 5)(x - 6) = 0$ .  
 $L + W + H = 15$ . **Ans. 15**

3. Critical points for  $|x^2 - 4| \geq |4x|$  are when (1)  $x^2 - 4 = 4x$  and (2)  $x^2 - 4 = -4x$ . In (1):

$$x^2 + 4x - 4 = 0 \rightarrow x = \frac{-4 \pm \sqrt{16 - 4(-4)}}{2} = \frac{-4 \pm \sqrt{32}}{2} = -2 \pm 2\sqrt{2}. \text{ In (2): } x^2 + 4x - 4 = 0 \rightarrow$$

$$\frac{4 \pm \sqrt{16 - 4(-4)}}{2} = \frac{4 \pm \sqrt{32}}{2} = 2 \pm 2\sqrt{2}. \text{ Since } \sqrt{2} \text{ is approximately } 1.4, \text{ then in (1): } -2 - 2\sqrt{2} \doteq -4.8,$$

$-2 + 2\sqrt{2} \doteq -0.2$ ,  $2 + 2\sqrt{2} \doteq 4.8$ , and  $2 - 2\sqrt{2} \doteq 0.2$ . Placing these on a number line:



Plugging in interval points: -5:  $21 \geq 20$ , yes;

-4.8:  $0 \geq 4$ , no; -0.2:  $4 \geq 0$ , yes; 0.2:  $0 \geq 8$ , no; 5:  $21 \geq 20$ , yes

$$\text{Ans. } x \leq -2 - 2\sqrt{2} \text{ or } 2 - 2\sqrt{2} \leq x \leq -2 + 2\sqrt{2} \text{ or } x \geq 2 + 2\sqrt{2}$$

### Individuals Round 4

1. Primes less than 24: 2, 3, 5, 7, 11, 13, 17, 19, 23. **Ans. (5, 19), (7, 17), (11, 13)**
2. Let  $C = x$ . Then  $A = 180 - 2x$ ,  $B = 90 - x$ ,  $C = x$ ,  $d = 4/5x \rightarrow 270 - \frac{6}{5}x = 180$ ,  $90 = \frac{6}{5}x$ .

$x = 75$ . So  $A = 180 - 150 = 30$ , and  $B = 90 - 75 = 15$ .  $A + B = 45$ .

**Ans. 45**

3.  $\sqrt{9x+12} - \sqrt{3x-2} = \sqrt{6x+2} \rightarrow 9x + 12 - 2\sqrt{9x+12}\sqrt{3x-2} + 3x - 2 = 6x + 2 \rightarrow$

$6x + 8 = 2\sqrt{9x+12}\sqrt{3x-2} \rightarrow 3x + 4 = \sqrt{27x^2 + 18x - 24} \rightarrow 9x^2 + 24x + 16 = 27x^2 + 18x - 24$

$0 = 3x^2 - 6x - 40 \rightarrow 0 = (3x + 4)(3x - 5)$ , so  $x = -\frac{4}{3}$  or  $\frac{5}{3}$ .  $-\frac{4}{3}$  does not work.

**Ans. 5/3**

### Individuals Round 5

1.  $\begin{vmatrix} 2 & -3 & 4 \\ 4 & 2 & -3 \\ 3 & 4 & -2 \end{vmatrix} = -8 + 27 + 64 - (24 - 24 + 24) = 83 - 24 = 59$ .

**Ans. 59**

2.  $x * y = 2x - y^2$  and  $x \# y = x/y$ .  $2 * 7 = 2(2) - 7^2 = -45$ .  $7 * 2 = 2(7) - 2^2 = 10$ .

$-45 \# 10 = -45/10 = -4.5$

**Ans. -4.5**

3.  $9(x^2 - 2x + 1) + 4(y^2 + 2y + 1) = 23 + 13 = 36$  or  $\frac{(x-1)^2}{4} + \frac{(y+1)^2}{9} = 1$  has center at  $(1, -1)$ . If moved 4 units to the right, the center would be  $(5, -1)$ . If the raised 5 units, the center would now be at  $(5, 4)$ . If now rotated  $90^\circ$  then the endpoints of the major axis would now end up on the horizontal axis:  $(5 \pm 3, 4) = (8, 4)$  and  $(2, 4)$ .

**Ans. (8, 4) and (2, 4)**

### Individuals Round 6

1.  $1.06x = 25.97$ ,  $x = 25.97/1.06 = 24.50$ . Tax =  $25.97 - 24.50 = \$1.47$

**Ans. \$1.47**

2.  $\frac{x^{a^2}}{x^{b^2}} = x^{24} \rightarrow x^{a^2-b^2} = x^{24}$ , thus  $(a-b)(a+b) = 24 \rightarrow a-b = \frac{24}{a+b} = \frac{24}{4} = 6$ .

**Ans. 6**

3.  $\log_5 6 + \log_{625} 2 + \log_{25} 3 = \log_5 N \rightarrow \log_5 6 + \log_5 2^{\frac{1}{4}} + \log_5 3^{\frac{1}{2}} = \log_5 6 \cdot 2^{\frac{1}{4}} \cdot 3^{\frac{1}{2}} =$

$\log_5 6\sqrt[4]{18}$ .

**Ans.  $6\sqrt[4]{18}$**

### Team Round 1

1.  $AP(PB) = CP(DP) \rightarrow 4(2/3x) = x(DP) \rightarrow 4(2/3) = DP = 8/3 = 2\frac{2}{3}$ .

**Ans.  $2\frac{2}{3}$**

2. Converting each to feet:  $\frac{1}{3} \left( \frac{29}{4} \right) \left( \frac{4}{3} \right) \left( \frac{19}{6} \right) = \frac{1}{3} \left( \frac{551}{18} \right) = \frac{1}{3} \left( 30\frac{11}{18} \right) = 10\frac{11}{54}$

**Ans.  $10\frac{11}{54}$**

3. Original bill: Cable + Internet + Telephone:  $.6x + .2x + .2x$ ,

new bill:  $1.25(.2x) + 1.5(.2x) + .2x = 250 \rightarrow 1.25x = 250$ , so  $x = 200$ .

**Ans. 200**

4. (1)  $x + y + 2z = 1$ , (2)  $3x - 4y - 5y = 4\frac{1}{2}$ , (3)  $4x + 3y + 2z = 2$ .  $-(1) + (3)$  and  $5(1) + 2(2)$ :

(4)  $3x + 2y = 1$  and (5)  $11x - 3y = 14$ .  $3(4) + 2(5)$ :  $9x + 6y = 3$  and  $22x - 6y = 28$ . Adding these two  $31x = 31$ , so  $x = 1$ . In (4):  $3(1) + 2y = 1$ , so  $y = -1$ . In (1):  $(1) + (-1) + 2z = 1$ ,

$$\text{so } z = \frac{1}{2}. \quad x + y + z = \frac{1}{2}.$$

**Ans. 1/2**

$$5. \frac{2x+3}{x+5} - \frac{3x+2}{x-5} + \frac{2x^2+13x+55}{x^2-25} \rightarrow \frac{(2x+3)(x-5) - (3x-2)(x+5) + 2x^2+13x+55}{(x-5)(x+5)} \rightarrow$$

$$\frac{2x^2-7x-15 - (3x^2+17x+10) + 2x^2+13x+55}{(x-5)(x+5)} \rightarrow \frac{x^2-11x+30}{(x-5)(x+5)} \rightarrow \frac{(x-5)(x-6)}{(x-5)(x+5)} \quad \text{Ans. } \frac{x-6}{x+5}$$

$$6. f(x) = \frac{4}{x-1} - \frac{x+3}{x^2-1} = \frac{4(x+1)}{(x-1)(x+1)} - \frac{x+3}{x^2-1} = \frac{4x+4-x-3}{x^2-1} = \frac{3x+1}{x^2-1}.$$

A function cannot cross a vertical asymptote, but it can cross a slant or horizontal one. So  $f(x) = \frac{3x+1}{x^2-1}$ , as  $x$  increases without bound,  $f(x)$  tends to 0.  $f(x) = 0$  is a horizontal asymptote. Setting these equal to each other tells where they intersect:  $\frac{3x+1}{x^2-1} = 0$ . So  $3x+1 = 0$ , or  $x = -1/3$ . The point is  $(-1/3, 0)$ .

**Ans. (-1/3, 0)**

7. In Series A: (1)  $345 = a + 20d$ ; (2)  $527 = a + 34d$ . (2)  $-$  (1):  $182 = 14d$ , so  $d = 13$ . In (1):

$$345 = a + 20(13) \rightarrow a = 345 - 260 = 85. \text{ So Series A} = 85 + 13d.$$

In Series B: (1)  $287 = a + 16d$  and (2)  $602 = a + 37d$ . (2)  $-$  (1):  $315 = 21d$ , so  $d = 15$ . In (1):

$$287 = a + 16(15) \rightarrow a = 287 - 240 = 47. \text{ So Series B} = 47 + 15d. \text{ This makes Series C equal to}$$

$$C = 132 + 28d. \text{ The sum of the first 38 terms: } 38^{\text{th}} \text{ term} = 132 + 28(37) = 132 + 1036 = 1068;$$

$$\text{Sum} = \frac{38(132+1068)}{2} = 19(1200) = 22800.$$

**Ans. 22,800**

$$8. \cos^2 \theta \csc \theta - \cot \theta = \sin \theta \rightarrow \frac{\cos^2 \theta}{\sin \theta} - \frac{\cos \theta}{\sin \theta} = \sin \theta \rightarrow \cos^2 \theta - \cos \theta = \sin^2 \theta \rightarrow$$

$\cos^2 \theta - \cos \theta = 1 - \cos^2 \theta \rightarrow 2\cos^2 \theta - \cos \theta - 1 = 0 \rightarrow (2\cos \theta + 1)(\cos \theta - 1) = 0$ . So  $\cos = 1$ , which is at  $0^\circ$  which cannot be used since  $\csc 0^\circ$  is undefined; or  $\cos \theta = -1/2$ , which is at  $120^\circ$  or  $240^\circ$ .

**Ans.  $120^\circ$  or  $240^\circ$**

### Team Round 2

$$1. a^{5/4} \cdot b^{1/6} \cdot c^{3/2} = a^{15/12} \cdot b^{2/12} \cdot c^{18/12} = \sqrt[12]{a^{15}b^2c^{18}}, \quad 12 + 15 + 2 + 18 = 47.$$

**Ans. 47**

2.  $FM = 3$ , since  $\triangle FCM$  is a 3-4-5 $\Delta$ . Therefore  $ME = CD = BC = 3$ . Thus the area of quadrilateral  $ABMF = 3(7) = 21$ , and the area of Quadrilateral  $CDEM = 12$ . **Ans. 33**

3.  $\frac{1}{16}(2+T-2) + \frac{1}{12}T = 1 \Rightarrow 3T + 4T = 48 \Rightarrow 7T = 48$ , so  $T = 6\frac{6}{7}$  hrs.  $\frac{6}{7}(60) = \frac{360}{7} = 51\frac{3}{7}$  min

Since Ben started at 8:00 AM, he finished at 2:51 PM to the closest minute. **Ans. 2:51 PM**

4.  $\log_{x-2}(x^3 - 6x^2 + 12x - 8)(\sqrt{x-2}) = n \Rightarrow x^3 - 6x^2 + 12x - 8 = (x-2)^3$  **Ans. 3 1/2**

5.  $\frac{2+3i}{4-i}(1+i) + (2-i)^{-1} \Rightarrow \frac{(2+3i)(1+i)}{4-i} + \frac{1}{2-i} \Rightarrow \frac{-1+5i}{4-i} + \frac{1(2+i)}{(2-i)(2+i)} \Rightarrow \frac{(-1+5i)(4+i)}{(4-i)(4+i)} + \frac{1(2+i)}{5}$

$\frac{-9+19i}{17} + \frac{2+i}{5} = \frac{5(-9+19i)+17(2+i)}{85} = \frac{-45+95i+34+17i}{85} = \frac{-11+112i}{85}$ .  $a + b = \frac{101}{85}$ . **Ans. 101/85**

6.  $f(x) = \frac{3}{1 - \frac{2}{x - \frac{1}{x-2}}}$ ,  $x \neq 2$ ;  $f(x) = \frac{3}{1 - \frac{2}{\frac{x^2 - 2x - 1}{x-2}}} = \frac{3}{1 - \frac{2x-4}{x^2 - 2x - 1}}$ ,  $x = \frac{2 \pm \sqrt{4 - 4(-1)}}{2} = 1 \pm \sqrt{2}$

So  $x \neq 1 \pm \sqrt{2}$ .  $f(x) = \frac{3}{\frac{x^2 - 2x - 1 - (2x - 4)}{x^2 - 2x - 1}} = \frac{3}{\frac{x^2 - 4x + 3}{x^2 - 2x - 1}} = \frac{3x^2 - 6x - 3}{x^2 - 4x + 3}$ ,  $x^2 - 4x + 3 = 0 \Rightarrow$

$(x-3)(x-1) = 0$ , so  $x \neq 1$  or  $3$ .

**Ans. All Reals except 1, 2, 3 or  $1 \pm \sqrt{2}$**

7. Using determinants for the area of  $\triangle ABC$  + area of  $\triangle ACD$ :

$$\frac{1}{2} \begin{vmatrix} -5 & 12 & 1 \\ -3 & -9 & 1 \\ 10 & -2 & 1 \end{vmatrix} = \frac{1}{2} (45 + 120 + 6 + 90 - 10 + 36) = \frac{1}{2} (287) = 143\frac{1}{2}$$

$$\frac{1}{2} \begin{vmatrix} -5 & 12 & 1 \\ 10 & -2 & 1 \\ 16 & 21 & 1 \end{vmatrix} = \frac{1}{2} (10 + 192 + 210 + 32 + 105 - 120) = \frac{1}{2} (429) = 214\frac{1}{2}$$

**Ans. 358**

8.  $\frac{x+2}{x+3} - \frac{x-2}{x+1} = \frac{2x+1}{5x-3} \rightarrow (x+2)(x+1)(5x-3) - (x-2)(x+3)(5x-3) = (2x+1)(x+3)(x+1) \rightarrow$   
 $(x^2+3x+2)(5x-3) - (x^2+x-6)(5x-3) = (2x+1)(x^2+4x+3) \rightarrow (2x+8)(5x-3) = 2x^3+9x^2+10x+3 \rightarrow$   
 $10x^2+34x-24 = 2x^3+9x^2+10x+3 \rightarrow 0 = 2x^3 - x^2 - 24x + 27.$

3  $\left| \begin{array}{cccc} 2 & -1 & -24 & 27 \\ & 6 & 15 & -27 \\ & & & \end{array} \right|$  So  $x = 3$  and for  $2x^2 + 5x - 9 = 0$ :  $x = \frac{-5 \pm \sqrt{25 - 4(-18)}}{4} = \frac{-5 \pm \sqrt{97}}{4}.$

$\left| \begin{array}{cccc} 2 & 5 & -9 & 0 \end{array} \right|$

**Ans. 3 or  $\frac{-5 \pm \sqrt{97}}{4}$**

**Blue Relay Seat A**

(1)  $8x - 5y = -3$  and (2)  $4x - 3y = -5$ : (1)  $-2(2)$ :  $-5y + 6y = -3 + 10$ ,  $y = 7$ . In (2):  $4x - 3(7) = -5$   
 $4x = 16$ , so  $x = 4$ .  $4 + 7 = 11$ . Pass:  $2A = 22$ .

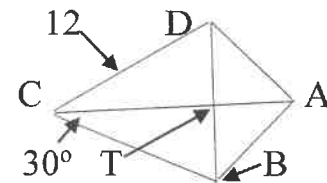
**Ans. A = 11, Pass: 22**

**Blue Relay Seat B**

Car A:  $80T = D$ . Car B:  $90T = D + 2$ .  $90T = 80T + 2$ ,  $10T = 2$ , so  $T = 1/5$ .  $80(1/5) = 16$ . So car A travelled 16 miles and car B 18 miles. Pass:  $\frac{X+B}{2} = \frac{22+18}{2} = 20$ . **Ans. B = 18, Pass: 20**

**Blue Relay Seat C**

In the kite at right  $\angle DAB$  is a right angle,  $AD = AB$ ,  $DC = BC$  and  $BC = 12$ .  $DB = 12$ ,  $CT = 6\sqrt{3}$  and  $AT = 6$ . Since  $\overline{AC} \perp \overline{BD}$  then the area is half the product of the diagonals:



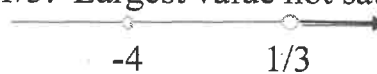
$Area = \frac{1}{2}(12)(6+6\sqrt{3}) = 36+36\sqrt{3}$ .  $m + p + q = 36 + 36 + 3 = 75$ .

Pass:  $1.5X + C = 1.5(20) + 75 = 30 + 75 = 105$

**Ans. C = 75, Pass: 105**

**Blue Relay Seat D**

$|2x-5| < 4x+3$ , critical points area where (1)  $2x - 5 = 4x + 3$  or (2)  $2x - 5 = -4x - 3$ . In (1):  
 $-8 = 2x$ , so  $x = -4$ . In (2):  $6x = 2$ , so  $x = 1/3$ . Plugging in interval points:  $-5$ :  $15 < -17$ , no;  
 $0$ :  $5 < 3$ , no;  $1$ :  $3 < 7$ , yes. So  $x < 1/3$ . Largest value not satisfying is  $1/3$ . Pass:  $DX - 2 =$   
 $(1/3)(105) - 2 = 35 - 2 = 33$ .



**Ans. D = 1/3, Pass: 33**

**Blue Relay Seat E**

2 pairs brown, 3 pairs blue, 3 pairs black. P(2 same color): 2 brown or 2 blue or 2 black  $\rightarrow$

$$\frac{{}_4C_2 + {}_6C_2 + {}_6C_2}{{}_{16}C_2} = \frac{6+15+15}{120} = \frac{36}{120} = \frac{3}{10}. \text{ Pass: } \frac{X}{E} = 33 \div \frac{3}{10} = 110. \text{ Ans. E} = 3/10, \text{ Pass: } 110$$

**Green Relay Seat A**

(1)  $9x + 7y = -3$ , (2)  $4x + 5y = 10$ .  $5(1) - 7(2): 45x - 28x = -15 - 70 \rightarrow 17x = -85, x = -5$ . In (1):  $9(-5) + 7y = -3, 7y = 42, y = 6$ .  $-5(6) = -30$ . Pass:  $-(1/2)(-30) = 15$ . **Ans. A = -30, Pass: 15**

**Green Relay Seat B**

Refer to Blue Seat B:  $80(1/5) = 16$ . Pass:  $2X - B = 2(15) - 16 = 14$ . **Ans. B = 16, Pass: 14**

**Green Relay Seat C**

Refer to Blue Seat C:  $\frac{1}{2}(6)(3+3\sqrt{3}) = 9+9\sqrt{3}$ .  $m + p + q = 21$ . Pass:  $2C - X = 2(21) - 14 = 28$ .

**Ans. C = 21, Pass: 28**

**Green Relay Seat D**

Looking at the graph of Blue Seat D, -4 is the smallest value that will not satisfy the ( $>$ ) part of the graph. Pass:  $X - 7D = 28 - 7(-4) = 56$ .

**Ans. D = -4, Pass: 56**

**Green Relay Seat E**

From Blue Seat E, the complement is  $7/10$ . Pass:  $\frac{X}{E} = 56 \cdot \frac{10}{7} = 80$ . **Ans. E = 7/10, Pass: 80**

**Pink Relay Seat A**

$$\frac{15x^3y^2 \cdot 16x^2y^5}{24x^4y^4} = ax^m y^n = 10xy^3, amn = 30. \text{ Pass: } \frac{1}{3}A = \frac{1}{3}(30) = 10. \text{ Ans. A} = 30, \text{ Pass: } 10$$

**Pink Relay Seat B**

$3.32 - x = \frac{1}{2}(4.48 + x) \rightarrow 6.64 - 2x = 4.48 + x \rightarrow 2.16 = 3x, \text{ so } x = .72$ .

Pass:  $10BX = 10(.72)(10) = 72$ .

**Ans. B = .72, Pass: 72**

**Pink Relay Seat C**

Since the 8-15-17 $\Delta$  is a right  $\Delta$ , its area is 60. The ratio of the areas of the two  $\Delta$ 's is  $\left(\frac{4}{3}\right)^2 = \frac{16}{9}$ .

Thus  $\frac{16}{9} = \frac{60}{x}, 16x = 60(9), x = \frac{15(9)}{4} = \frac{135}{4} = 33\frac{3}{4}$ . Pass:  $4C - X: 4\left(\frac{135}{4}\right) - 72 = 63$ .

**Ans. C =  $33\frac{3}{4}$ , Pass: 63**

**Pink Relay Seat D**

$$\begin{bmatrix} 4 & 3 \\ 2 & -1 \\ 5 & 4 \end{bmatrix} \cdot \begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 32 \\ -7 & 6 \\ 2 & 41 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \\ e & f \end{bmatrix}. \quad a + b + c + d + e + f = 75. \quad \text{Pass: } X - \frac{D}{3} = 63 - \frac{75}{3} = 38.$$

**Ans. D = 75, Pass: 38****Pink Relay Seat E**

$$\tan^2 \theta + \sin^2 \theta = 2 - \cos^2 \theta \rightarrow \tan^2 \theta + \sin^2 \theta + \cos^2 \theta = 2 \rightarrow \tan^2 \theta + 1 = 2 \rightarrow \tan^2 \theta = 1 \rightarrow$$

$$\tan \theta = \pm 1, \text{ thus } \theta = 45, 135, 225, 315. \quad 315 - 45 = 270. \quad \text{Pass: } X + \frac{E}{10} = 38 + \frac{270}{10} = 65.$$

**Ans. E = 270, Pass: 65****Yellow Relay Seat A**

$$\frac{30x^5y^7 \cdot 24x^2y^4}{36x^3y^3} = 20x^4y^8 = ax^m y^n, \quad \frac{am}{n} = \frac{20 \cdot 4}{8} = 10. \quad \text{Pass: } 2A = 20. \quad \text{Ans. A = 10, Pass: 20}$$

**Yellow Relay Seat B**

$$3.50 - x = (1/5)(4.30 + x) \rightarrow 17.50 - 5x = 4.30 + x \rightarrow 13.20 = 6x, \text{ so } x = 2.20.$$

$$\text{Pass: } BX = (2.20)(20) = 44.$$

**Ans. B = 2.20, Pass: 44****Yellow Relay Seat C**

$$\text{The } 9\text{-}12\text{-}21\Delta \text{ has a perimeter of } 42. \text{ So } \frac{12}{42} = \frac{30}{x}, \quad 12x = 30(42) \rightarrow x = 5(21) = 105.$$

$$\text{Pass: } \frac{4C + 20}{X} = \frac{4(105) + 20}{44} = \frac{420 + 20}{44} = 10.$$

**Ans. C = 105, Pass: 10****Yellow Relay Seat D**

$$\text{From Pink Seat D: } 1 + 32 - 7 - (6 + 2 + 41) = 26 - 49 = -23. \quad \text{Pass: } 2X - D = 2(10) - (-23) = 20 + 23 = 43.$$

**Ans. D = -23, Pass: 43****Yellow Relay Seat E**

$$\tan^2 \theta + \cos^2 \theta = 4 - \sin^2 \theta, \text{ as in Pink Seat E, } \tan^2 \theta = 3 \rightarrow \tan \theta = \pm\sqrt{3}, \text{ therefore}$$

$$\theta = 60, 120, 240, 300. \quad 300 - 60 = 240. \quad \text{Pass: } X - \frac{E}{10} = 43 - \frac{240}{10} = 19. \quad \text{Ans. E = 240, Pass: 19}$$



## Answer Sheet – States 2017

### Individuals Round 1

1. 22

2.  $1/3$

3. 12

### Individuals Round 2

1.  $\frac{b+1}{c-a}$

2.  $36\sqrt{3}$

3.  $-5\frac{5}{8}$  or  $-45/8$

### Individuals Round 3

1. 21

2. 15

3.  $x \leq -2 - 2\sqrt{2}$  or  $2 - 2\sqrt{2} \leq x \leq -2 + 2\sqrt{2}$  or  $x \geq 2 + 2\sqrt{2}$

### Individuals Round 4

1. (5, 19), (7, 17), (11, 13)

2. 45 or  $45^\circ$

3.  $5/3$

### Individuals Round 5

1. 59

2.  $-4\frac{1}{2}$  or  $-9/2$

3. (8, 4) and (2, 4)

### Individuals Round 6

1. 1.47 or \$1.47

2. 6

3.  $6\sqrt[4]{18}$

### Team Round 1

1.  $2\frac{2}{3}$  or  $8/3$

2.  $10\frac{11}{54}$  or  $551/54$

3. 200 or \$200

4.  $1/2$

5.  $\frac{x-6}{x+5}$

6.  $(-1/3, 0)$

7. 24,700

8. 120, 240 or  $120^\circ, 240^\circ$

### Team Round 2

1. 47

7. 358

2. 33

8.  $\frac{-5 \pm \sqrt{97}}{4}$  or 3

3. 2:51 PM

4.  $3\frac{1}{2}$  or  $7/2$  or 3.5

5.  $101/85$

6. All Reals  $\neq 1 \pm \sqrt{2}, 1,$

### Relays

2 or 3

Blue	Ans.	Pass	Green	Ans.	Pass	Pink	Ans.	Pass	Yellow	Ans.	Pass
A	11	22	A	-30	15	A	30	10	A	10	20
B	18	20	B	16	14	B	.72	72	B	2.20	44
C	75	105	C	21	28	C	$33\frac{3}{4}$	63	C	105	10
D	$1/3$	33	D	-4	56	D	75	38	D	-23	43
E	$3/10$	110	E	$7/10$	80	E	270	65	E	240	19

