

Individuals – Round 1 States 2019

3 pts 1. From what polynomial must $5x^3 - 2x^2 + 3x$ be subtracted in order to obtain $x^3 - 2$?

Ans. _____

4 pts 2. Simplify: $\frac{80 - 3[2^3 - (-4 - (-1)) + 3^2]}{-8 \div 4 \cdot 2}$.

Ans. _____

5 pts 3. Find the sum of the integral solutions to $|x - 45| < 23$.

Ans. _____

Individuals – Round 2 States 2019

3 pts 1. A line crosses the y-axis at $(a + 2010, a + 2011)$. What is the value of a ?

Ans. _____

4 pts 2. Points L and U lie on the segment KN. $\frac{LK}{LN} = \frac{4}{1}$ and $\frac{UN}{UK} = \frac{5}{3}$. If the fully reduced ratio of the length of LU to KN is $p:q$, where p and q are relatively prime, what is the numerical value of $p + q$?

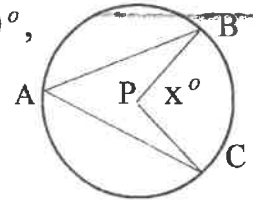
Ans. _____

5 pts 3. What is the maximum possible value of $\frac{|7 + 24i|}{|a + 4i|}$, where a is a real number?

Ans. _____

Individuals – Round 3 States 2019

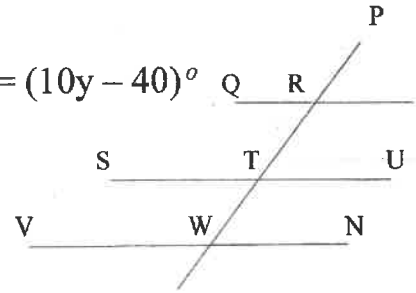
3 pts 1. Point P is the center of the circle, $m\angle ABP = 30^\circ$, $m\angle ACP = 20^\circ$, and $m\angle BPC = x^\circ$. Find the value of x.



Ans. _____

4 pts 2. Given $\overline{QR} \parallel \overline{ST} \parallel \overline{VW}$, $m\angle PRQ = (8x + 14)^\circ$, $m\angle UTW = (10y - 40)^\circ$

And $m\angle TWN = (3x + y + 19)^\circ$. Find $m\angle QRT - m\angle RTS$.



Ans. _____

5 pts 3. Let N be the smallest positive integer which ends with the number 17, which is divisible by 17 and whose digits have a sum of 17. Find the product of the digits of N.

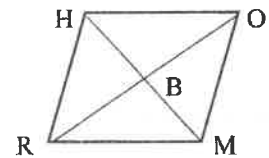
Ans. _____

Individuals – Round 4 States 2019

3 pts 1. A math-magic trick using natural numbers is performed as follows: pick any number, add 5, multiply this answer by 6, subtract 15 from this new answer, divide this answer by 3, subtract this answer by the original number, then perform one final operation on this answer. What should the final operation be so that the result of this trick is always the original number? Write the operation and the number.

Ans. _____

4 pts 2. RHOM is a rhombus with $RH = x + 2$, $BH = x$ and $BO = x + 1$. Find the area of Quadrilateral RHOM.



Ans. _____

5 pts 3. If $\frac{x^a}{x^b} = x^{28}$, $x > 1$ and $a + b = 4$, what is the value of $a - b$?

Ans. _____

Individuals – Round 5 States 2019

3 pts 1. Let a be defined as $a * b = ab - b^a$. Find the value of $4 * (2 * 3)$.

Ans. _____

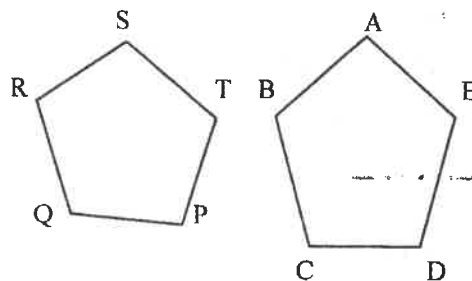
4 pts 2. For real numbers a , b , and c , $(ax + 3)(bx + 2) = 15x^2 + cx + 6$ and $a + b = 16$. Find 2 possible values of c .

Ans. _____

5 pts 3. Pentagon ABCDE is similar to pentagon PTSRQ.

$AB = 20$, $BC = 24$, $CD = 18$, $DE = 25$, $AE = 22$, and $RS = 20$.

Find the perimeter of pentagon PTSRQ.



Ans _____

Individuals – Round 6 States 2019

3 pts 1. Two sides of an isosceles triangle are 4 and 12. What is the perimeter of the triangle?

Ans. _____

4 pts 2. The graph of $\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$ has asymptotes with equations $2x + 3y = -4$ and $2x - 3y = 8$. Find the value of $h - k$.

Ans. _____

5 pts 3. Find all values of x for which $\log_{13}(x^3 - 1) + \log_{\frac{1}{13}}(x - 1) = 1$.

Ans. _____

Team – Round 1 States 2019

4 pts 1. $\frac{(xy-z)(xy+z)+z^2}{yz^3}$ can be written in the form $x^A y^B z^C$.

Find the value of $A + B - C$.

(1) Ans. _____ **4 pts**

4 pts 2. Find the sum of the negative integers which are in the solution set for the inequality

$$8 - \frac{n}{3} > 1 - 2(3 + n).$$

(2) Ans. _____ **4 pts**

6 pts 3. In a game the score is obtained by subtracting the number of incorrect answers from twice the number of correct answers. If Mark answered 40 questions and obtained a score of 50, how many questions did Mark answer correctly.

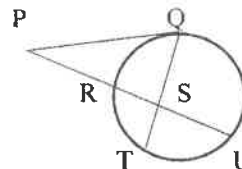
(3) Ans. _____ **6 pts**

6 pts 4. The Islesford School District formed a committee of 15% parents, 45% teachers, 25% school and district administrators and the remaining 6 are students. How many more teachers were on the committee than school and district administrators?

(4) Ans. _____ **6 pts**

6 pts 5. \overline{PQ} is tangent to the circle at Q and 6 units long.

$RS = 2$, $PR = 3$, $ST = 4$, $QS = b$ and $SU = a$. Find the value of $4a - 2b$.



(5) Ans. _____ **6 pts**

8 pts 6. How many distinct ordered pairs of positive integers (x, y) satisfy $x^y = 2^{20}$.

(6) Ans. _____ **8 pts**

8 pts 7. The sum of the first n positive integers is 150 less than the sum of the first $3n$ positive integers. What is the sum of the first $5n$ positive integers?

(7) Ans. _____ **8 pts**

8 pts 8. Find the sum of all values of x , where $0^\circ \leq x < 360^\circ$, for which

$$\sec^2 x + 2 \cos x \sin x = 2 + \tan^2 x.$$

(8) Ans. _____ **8 pts**

Team – Round 2 States 2019

4 pts 1. Express $4(a^2 - 2b + 5) - 3(4a^2 + b - 7)$ in terms of b , if $a = -2b$.

(1) Ans. _____ 4 pts

4 pts 2. In a large room, the ratio of 8th graders to 9th graders is 4 to 3. The ratio of 10th graders to 11th graders is 3 to 2. The ratio of 11th graders to 9th graders is 1 to 6. What is the ratio of 8th graders to 10th graders?

(2) Ans. _____ 4 pts

6 pts 3. Solve for x , if $\frac{x^2 - 3x - 10}{x^2 + 3x - 28} \cdot \frac{x^2 + 8x + 7}{x^2 - x + 20} = 1$.

(3) Ans. _____ 6 pts

6 pts 4. There are 6 males and 4 females on a committee. 3 are chosen at random. What is the probability that 2 males and 1 female are ones chosen?

(4) Ans. _____ 6 pts

6 pts 5. A grain silo is built in the shape of a right-circular cylinder with two right-circular cones attached, one on the top and one on the bottom. The height of the cylinder is 10 ft. and the total height of the silo is 20 ft. The radius of the cylinder is 5 ft. Find the volume of the silo.

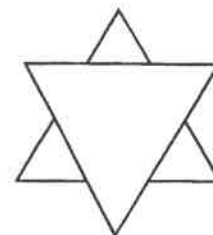
(5) Ans. _____ 6 pts

8 pts 6. Find the number of ordered pairs (a, b) of positive integers

for which $2a + b \leq 9$.

(6) Ans. _____ 8 pts

8 pts 7. Two congruent equilateral triangles, whose perimeters each equal 36, overlap to form a 6-pointed star. All six petals of the star are congruent. What is the area of the concave dodecagon?



(7) Ans. _____ 8 pts

8 pts 8. Find all real values of x , such that $\begin{vmatrix} x & 5 & x+3 \\ 3 & x-2 & 6 \\ -5 & -x & x-3 \end{vmatrix} = -75$.

(8) _____ 8 pts

Seat A Blue Relay States 2019

If $a = b^2 - c$, $b = \frac{c}{2} + 1$ and $c = -3^3 + 23$, find the numerical value of $a + b$.

Pass back: $2A^2$

A = Your answer

Seat B Blue Relay

I have \$17 consisting of silver dollars and quarters. What is the value of my quarters in dollars, if I have 26 coins in all?

Pass back: $B + X + 2$

B = Your answer

X = The number you will receive

Seat C Blue Relay

An angle's complement is 54° less than eight times the angle. What is the value of the angle's complement in degrees?

Pass back: $\sqrt{\frac{32C}{X}}$

C = Your answer

X = The number you will receive

Seat D Blue Relay

If $\log_3 6 = x$, what is the value of 9^{2x-1} ?

Pass back: $D - 2X$

D = Your answer

X = The number you will receive

Seat E Blue Relay

For two acute angles a and b , $\sin a = \cos b$. If $a = 4k - 22$ and $b = 6k - 13$, what is the value of $10k$?

Pass in: $(X - E)^2$

E = Your answer

X = The number you will receive

Seat A Green Relay States 2019

If $a = b^2 - c$, $b = \frac{c}{2} + 1$ and $c = -5^2 + 19$, find the value of $a + b$.

Pass back: $13A + 1$

A = Your answer

Seat B Green Relay

In my coin dish I have 90 coins consisting of only dimes and quarters. If there is \$18 in my coin dish, what is the value in dollars of all my quarters?

Pass back: $\frac{X}{B}$

B = Your answer

X = The number you will receive

Seat C Green Relay

An angle's supplement is 36° more than seven times the angle. What is the measure of the angle?

Pass back: $1000(C + X)$

C = Your answer

X = The number you will receive

Seat D Green Relay

If $\log_3 10 = x$, what is the value of 27^{x+1} ?

Pass back: $\frac{D-X}{400}$

D = Your answer

X = The number you will receive

Seat E Green Relay

For two acute angles a and b , $\sin a = \cos b$. If $a = 7k - 21$ and $b = 4k + 12$, what is the value of k ?

Pass in: $X^{(E-X)}$

E = Your Answer

X = The number you will receive

Seat A Pink Relay States 2019

Gena reduced $\sqrt{605}$ correctly to $a\sqrt{b}$. However, she wrote her answer as $b\sqrt{a}$. Find c , if $b\sqrt{a} = \sqrt{c}$.

Pass back: A – 19

A = Your answer

Seat B Pink Relay

The sum of the numerator and denominator of a certain fraction is 27. If 4 is subtracted from the numerator and 5 is added to the denominator, the fraction will equal $\frac{1}{3}$, when reduced. Find the original fraction.

Pass back: BX – 66

B = Your answer

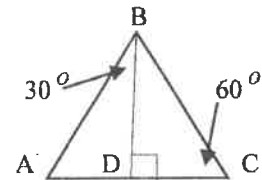
X = The number you will receive

Seat C Pink Relay

In triangle ABC, find $(BD)^2$. $\overline{BD} \perp \overline{AC}$ and $BC = 12$.

Pass back: $(X - C)^5$

C = Your answer



X = The number you will receive

Seat D Pink Relay

The minimum value of the function $f(x) = 2x^2 - 8x + k$ is 24. Find the value of k .

Pass back: $2\sqrt{DX}$

D = Your answer

X = The number you will receive

Seat E Pink Relay

$$\begin{bmatrix} \frac{x}{2} & 1 & 1 \\ 2 & 2 & 3 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 \\ 2 & 3 \\ 2 & -2 \end{bmatrix} = M \text{ and } |M| = -16. \text{ Solve for } x.$$

Pass in: $7\sqrt[4]{X}$

E = Your answer

X = The number you will receive

Seat A Yellow Relay States 2019

Gene reduced $\sqrt{245}$ correctly as $a\sqrt{b}$. However, he wrote his answer incorrectly as $b\sqrt{a}$.

If $b\sqrt{a} = \sqrt{c}$, find c .

Pass back: A - 15 A = Your answer

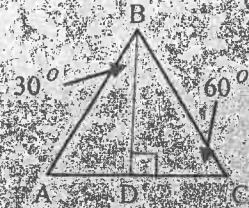
Seat B Yellow Relay

The sum of the numerator and denominator of a certain fraction is 47. If 2 is subtracted from the denominator and 5 is added to the numerator, the fraction will equal $\frac{2}{3}$, when reduced. Find the original fraction.

Pass back: BX B = Your answer X = The number you will receive

Seat C Yellow Relay

In $\triangle ABC$, $\overline{BD} \perp \overline{AC}$, $BD = 6\sqrt{3}$. Find the perimeter of $\triangle ABC$.



Pass back: $\left(\frac{X}{\sqrt{C+9}}\right)^2$ C = Your answer

X = The number you will receive

Seat D Yellow Relay

The minimum value of the function $f(x) = 2x^2 + 8x + k$ is -8. What is the value of k ?

Pass back: $D + X + 2$ D = Your answer X = The number you will receive

Seat E Yellow Relay

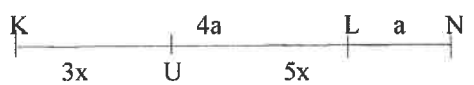
$\begin{bmatrix} \frac{x}{2} & 1 & 3 \\ -3 & 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 & 1 \\ 3 & 2 \\ -2 & 2 \end{bmatrix} = M$ and $|M| = 23$. Solve for x .

Pass in: $\left(\frac{X}{E-1}\right)^4$ E = Your answer X = The number you will receive

Solutions – Individuals Round 1

1. $(5x^3 - 2x^2 + 3x) + (x^3 - 2) = 6x^3 - 2x^2 + 3x - 2$. **Ans. $6x^3 - 2x^2 + 3x - 2$**
2. $\frac{80 - 3[2^3 - (-4 - (-1)) + 3^2]}{-8 \div 4 \cdot 2} = \frac{80 - 3[8 + 3 + 9]}{-2 \cdot 2} = \frac{20}{-4} = -5$. **Ans. - 5**
3. (1) $x - 45 < 23$ and (2) $-x + 45 < 23$. In (1) $x < 68$. In (2) $x > 22$. Thus the sum of the whole numbers from 23 to 67: $45\left(\frac{23+67}{2}\right) = 45(45) = 2025$. **Ans. 2025**

Individuals Round 2

1. A line crosses the y-axis at $x = 0$. Thus $a + 2010 = 0$, so $a = -2010$. **Ans. - 2010**
2. The 4:1 ratio and the 5:3 ratio have totals of 5 and 8. So using the LCM of 40. $4a + a = 40$, and $5x + 3x = 40$. $a = 8$, and $b = 5$. So $LU:KN = (40 - 8 - 15):40 = 17:40$. $p + q = 57$. **Ans. 57**
- 
3. $\frac{|7+24i|}{|a+4i|} = \frac{\sqrt{7^2+24^2}}{\sqrt{a^2+4^2}} = \frac{25}{\sqrt{a^2+16}}$. The denominator is smallest when $a = 0$. **Ans. 25/4**

Individuals Round 3

1. Connect A to P. $\angle PAB = 30$, $\angle CAP = 20$. So $\angle BAC = 50$, thus $\angle BC = 100 = x$. **Ans. 100**
2. $8x + 14 = 10y - 40 \rightarrow (1) 4x - 5y = -27$. $10y - 40 + 3x + y + 19 = 180 \rightarrow (2) 3x + 11y = 201$. $3(1) - 4(2) = (12x - 15y = -81) + (-12x - 44y = -804) \rightarrow -59y = -885$, $y = 15$. In (1): $4x - 5(15) = -27 \rightarrow 4x = 48$, $x = 12$. $8x + 14 = 8(12) + 14 = 110 = m\angle QRP$. $m\angle QRT - m\angle RTS = 70 - 110 = -40$. **Ans. - 40**
3. 17 produces 8 as sum of digits so far. The rest of the numbers have to add to 9 and be the smallest number. Thus starting with 17, then 34 etc. we are looking for multiples of 17 whose digits add to 9. The smallest is 153. The number is 15317. Multiplying the digits = **Ans. 105**

Individuals Round 4

1. I picked 9: $9 + 5 = 14 \times 6 = 84 - 15 = 69 / 3 = 23 - 9 = 14$. 14 subtract 5 = 9. Students should try more than one number to prove the answer. **Ans. Subtract 5**
2. By Pyth. Thm: $(x+2)^2 = x^2 + (x+1)^2 \rightarrow x^2 + 4x + 4 = x^2 + x^2 + 2x + 1 \rightarrow 0 = x^2 - 2x - 3 \rightarrow 0 = (x-3)(x+1)$. $x = 3$. Diagonals are 6 and 8. Area = $\frac{1}{2}(48)$. **Ans. 24**

$$3. \frac{x^{a^2}}{x^{b^2}} = x^{28} \rightarrow x^{a^2} \cdot x^{-b^2} = x^{28}, \text{ thus } a^2 - b^2 = 28, a^2 - (4-a)^2 = 28 \rightarrow a^2 - 16 + 8a - a^2 = 28,$$

$$8a = 44, \text{ so } a = 5\frac{1}{2} \text{ and } b = -1\frac{1}{2}. a - b = 5\frac{1}{2} - (-1\frac{1}{2}) = 7. \text{ Alt. sol.: } a^2 - b^2 = 28, \text{ so}$$

$$(a+b)(a-b) = 28 \rightarrow 4(a-b) = 28, \text{ then } a-b = 7.$$

Ans. 7

Individuals Round 5

$$1. 2 * 3 = 2(3) - 3^2 = -3. 4 * (-3) = 4(-3) - (-3)^4 = -12 - 81 = -93.$$

Ans. -93

$$2. (ax+3)(bx+2) = 15x^2 + cx + 6 \rightarrow abx + 3bx + 2ax + 6 = 15x^2 + cx + 6. \text{ Thus } ab = 15$$

and since $a+b=16$, then either $a=15$ and $b=1$, or $a=1$ and $b=15$. $3a+2b=c$, then

$$c = 3(15) + 2(1) = 47 \text{ or } 3(1) + 2(15) = 33.$$

Ans. 47, 33

$$3. \frac{PQ}{RS} = \frac{AE}{CD} \rightarrow \frac{m}{20} = \frac{22}{18}, \frac{ST}{RS} = \frac{BC}{CD} \rightarrow \frac{x}{20} = \frac{24}{18}, \frac{PT}{RS} = \frac{AB}{CD} \rightarrow \frac{y}{20} = \frac{20}{18}, \frac{DE}{CD} = \frac{RQ}{RS} \rightarrow \frac{n}{20} = \frac{25}{18}.$$

$$\text{Thus } \frac{18}{20}(m+x+y+n) = 22 + 24 + 20 + 25 \rightarrow m+x+y+n = \frac{10}{9}(91) = \frac{910}{9} = 101\frac{1}{9}. \text{ This}$$

$$\text{added to } 20 \text{ makes } 121\frac{1}{9}.$$

Ans. 121 $\frac{1}{9}$

Individuals Round 6

$$1. \text{ The other side has to be } 12. \text{ Perimeter: } 4 + 12 + 12 = 28.$$

Ans. 28

$$2. (1) 2x + 3y = -4 \text{ and } (2) 2x - 3y = 8. (1) + (2): 4x = 4, x = 1. \text{ In } (1): 2(1) + 3y = -4,$$

$$3y = -6, y = -2. (1, -2) = (h, k), h - k = 1 - (-2) = 3.$$

Ans. 3

$$3. \log_{13}(x^3-1) + \log_{\frac{1}{13}}(x-1) = 1 \rightarrow \log_{13}(x^3-1) + \log_{13}\left(\frac{1}{x-1}\right) = 1 \rightarrow \log_{13}\left(\frac{x^3-1}{x-1}\right) = 1 \rightarrow$$

$$x^3 - 1 = 13x - 13 \rightarrow x^3 - 13x + 12 = 0 \rightarrow (x-3)(x^2 + 3x - 4) = 0 \rightarrow (x-3)(x+4)(x-1) = 0.$$

$$\text{So } x = 3, -4, 1. x \text{ cannot } = 1 \text{ or } -4.$$

Ans. 3

Team Round 1

$$1. \frac{(xy-z)(xy+z)+z^2}{yz^3} = \frac{x^2y^2 - z^2 + z^2}{yz^3} = \frac{x^2y^2}{yz^3} = \frac{x^2y}{z^3} = x^2y^1z^{-3}. A+B-C = 6. \text{ Ans. 6}$$

$$2. 8 - \frac{n}{3} > 1 - 2(3+n) \rightarrow 24 - n > 3 - 18 - 6n \rightarrow 5n > -39, n > -7\frac{4}{5}. \text{ The negative numbers}$$

$$\text{are from } -7 \text{ to } -1. \text{ So } -\frac{7(8)}{2} = -28.$$

Ans. -28

$$3. \text{ So } 2C - N = \text{score. Let } x = \# \text{ incorrect answers. Then } 2(40-x) - x = 50, 80 - 3x = 50 \rightarrow$$

$$30 = 3x, 10 \text{ incorrect answers, so } 30 \text{ correct answers.}$$

Ans. 30

4. 85% are non-students. 15% are student. $.15x = 6$, $x = 40$ committee members. $45\%x - 25\%x = 18 - 10 = 8$. **Ans. 8**

5. $PQ^2 = PR(PU) \rightarrow 36 = 3(3 + RU) \rightarrow 12 = 3 + RU$, $RU = 9$, so $SU = 7 = a$.
 $QS(ST) = RS(SU)$, so $b(4) = 2(7)$, $b = 3\frac{1}{2}$. $4a - 2b = 4(7) - 2(3\frac{1}{2}) = 21$. **Ans. 21**

6. $x^y = 2^{20} = 4^{10} = 16^5 = 32^4 = 1024^2 = (20^2)^1$ **Ans. 6**

7. $\frac{3n(3n+1)}{2} - \frac{n(n+1)}{2} = 150 \rightarrow 9n^2 + 3n - n^2 - n = 300 \rightarrow 8n^2 + 2n - 300 = 0$.

$4n^2 + n - 150 = 0 \rightarrow (4n + 25)(n - 6) = 0$, $n = 6$. $5n = 30$. $\text{Sum} = \frac{30(31)}{2} = 465$. **Ans. 465**

8. $\sec^2 x + 2 \cos x \sin x = 2 + \tan^2 x \rightarrow \sec^2 x - \tan^2 x + 2 \cos x \sin x = 2 \rightarrow$

$1 + \sin 2x = 2 \rightarrow \sin 2x = 1$. $2x = 90^\circ$, so $x = 45^\circ$. Since $\sin 2x$ repeats every 180° , then $x = 225^\circ$ also. Thus $45^\circ + 225^\circ = 270^\circ$. **Ans. 270°**

Team Round 2

1. $4(a^2 - 2b + 5) - 3(4a^2 + b - 7) \rightarrow 4a^2 - 8b + 20 - 12a^2 - 3b + 21$. Since $a = -2b$, then
 $4(-2b)^2 - 8b + 20 - 12(-2b)^2 - 3b + 21 \rightarrow 16b^2 - 48b^2 - 11b + 41$. **Ans. $-32b^2 - 11b + 41$**

2. $\frac{8th}{9th} = \frac{4}{3}$, $\frac{9th}{11th} = \frac{6}{1}$, $\frac{11th}{10th} = \frac{2}{3} \rightarrow \frac{4}{3} \cdot \frac{6}{1} \cdot \frac{2}{3} = \frac{16}{3}$. **Ans. 16/3**

3. $\frac{x^2 - 3x - 10}{x^2 + 3x - 28} \cdot \frac{x^2 + 8x + 7}{x^2 - x + 20} = 1 \rightarrow \frac{(x-5)(x+2)}{(x+7)(x-4)} \cdot \frac{(x+7)(x+1)}{(x-5)(x+4)} = 1 \rightarrow x^2 + 3x + 2 = x^2 - 16 \rightarrow$
 $3x = -18$, $x = -6$. **Ans. -6**

4. $\frac{{}_6C_2 \cdot {}_4C_1}{{}_{10}C_3} = \frac{15 \cdot 4}{10 \cdot 9 \cdot 8} = \frac{15 \cdot 4}{10 \cdot 3 \cdot 4} = \frac{1}{2}$. **Ans. 1/2**

5. Cylinder + 2 Cones: $\pi 5^2(10) + 2(\frac{1}{3})\pi 5^2(5) = 250\pi + \frac{2}{3}(125)\pi = \frac{1000\pi}{3}$. **Ans. $\frac{1000\pi}{3}$**

6. (1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (3, 1),
(3, 2), (3, 3), (4, 1). There are 16. **Ans. 16**

7. Each small triangle is an equilateral triangle with 4 unit length sides. So each triangle has an area of $4\sqrt{3}$. The inner hexagon can be divided into 6 congruent triangles. All 12 triangles are congruent and have the same area. $12(4\sqrt{3}) = 48\sqrt{3}$. **Ans. $48\sqrt{3}$**

$$8. x(x-2)(x-3) - 150 - 3x(x+3) + 5(x-2)(x+3) + 6x^2 - 15(x-3) = -75 \rightarrow$$

$$x^3 - 5x^2 + 6x - 150 - 3x^2 - 9x + 5x^2 + 5x - 30 + 6x^2 - 15x + 45 = -75 \rightarrow x^3 + 3x^2 - 13x - 60 = 0 \rightarrow (x-4)(x^2 + 7x + 15). \quad x = 4, \text{ discriminant: } \sqrt{49 - 4(15)} = \sqrt{-11}, \text{ no solution.} \quad \text{Ans. 4}$$

Seat A Blue Relay

$$c = -27 + 23 = -4, b = -4/2 + 1 = -1, a = (-1)(-1) - (-4) = 5. \quad A + b = 5 + (-1) = 4.$$

$$2A^2 = 2(4)^2 = 32.$$

$$\text{A} = 4, \text{ Pass } 32$$

Seat B Blue Relay

$$25x + 100(26 - x) = 1700 \rightarrow 25x + 2600 - 100x = 1700 \rightarrow -75x = -900, x = 12. \quad 12 \text{ quarters.}$$

$$\text{That makes 3 dollars. Pass back: } B + X + 2 = 3 + 32 + 2 = 37.$$

$$\text{B} = 3, \text{ Pass } 37$$

Seat C Blue Relay

$$90 - x = 8x - 54 \rightarrow 144 = 9x, x = 16. \quad \text{Comp} = 74. \quad \text{Pass: } \sqrt{\frac{32C}{x}} = \sqrt{\frac{32(74)}{37}} = 8. \quad \text{C} = 74, \text{ Pass } 8$$

Seat D Blue Relay

$$\log_3 6 = x, \text{ so } 3^x = 6. \quad 9^{2x-1} = 9^{2x}/9 = 3^{4x}/9 = (3^x)^4/9 = 6^4/9 = 144. \quad \text{Pass back: } D - 2X =$$

$$144 - 2(8) = 144 - 16 = 128.$$

$$\text{D} = 144, \text{ Pass } 128$$

Seat E Blue Relay

$$\text{If } \sin a = \cos b, \text{ then } a \text{ and } b \text{ are complementary. } 4x - 22 + 6x - 13 = 90 \rightarrow 10x - 35 = 90 \rightarrow$$

$$10x = 125 = E. \quad \text{Pass in: } (X - E)^2 = (128 - 125)^2 = 9.$$

$$\text{E} = 125, \text{ Pass } 9$$

Seat A Green Relay

$$c = -25 + 19 = -6, b = -6/2 + 1 = -2, a = (-2)^2 + 6 = 10. \quad a + b = 8.$$

$$\text{A} = 8, \text{ Pass } 105$$

Seat B Green Relay

$$10(90 - x) + 25x = 1800 \rightarrow 900 - 10x + 25x = 1800 \rightarrow 15x = 900, \rightarrow x = 60. \quad 60 \text{ quarters}$$

$$\text{makes 15 dollars. Pass back: } X/B = 105/15 = 7.$$

$$\text{B} = 15, \text{ Pass } 7$$

Seat C Green Relay

$$180 - x = 7x + 36 \rightarrow 144 = 8x, x = 18. \quad \text{Pass back: } 1000(C + X) = 25000. \quad \text{C} = 18, \text{ Pass } 25,000$$

Seat D Green Relay

$$3^x = 10. \quad 27^{x+1} = 27(27)^x = 27(3^3)^x = 27(3^x)^3 = 27(10)^3 = 27,000. \quad \text{Pass back: } \frac{D - X}{400} =$$

$$\frac{27000 - 25000}{400} = \frac{2000}{400} = 5.$$

$$\text{D} = 27,000, \text{ Pass } 5$$

Seat E Green Relay

$$7k - 21 + 4k + 12 = 90, 11k = 99, x = 9. \text{ Pass in: } X^{(E-X)} = 5^4 = 625. \quad \mathbf{E = 9, Pass 625}$$

Seat A Pink Relay

$$\sqrt{605} = 11\sqrt{5}. 5\sqrt{11} = \sqrt{c}, c = 25(11) = 275. \text{ Pass: } A - 19 = 256. \quad \mathbf{A = 275, Pass 256}$$

Seat B Pink Relay

$$\frac{x-4}{27-x+5} = \frac{1}{3} \rightarrow 3x - 12 = 32 - x \rightarrow 4x = 44, x = 11. \text{ Orig. Frac.} = \frac{11}{16}.$$

$$\text{Pass back: } BX - 66 = \left(\frac{11}{16}\right)(256) - 66 = 11(16) - 66 = 176 - 66 = 110. \quad \mathbf{B = 11/16, Pass 110}$$

Seat C Pink Relay

$$DC = 6, BD = 6\sqrt{3}. (6\sqrt{3})^2 = 36(3) = 108. \text{ Pass back: } (110 - 108)^5 = 32. \quad \mathbf{A = 108, Pass 32}$$

Seat D Pink Relay

$$\text{x-coordinate of vertex is at } \frac{-b}{2a} = \frac{-(-8)}{2(2)} = 2. \text{ Vertex: } (2, 24). \text{ Plugging in: } 2(2)^2 - 8(2) + k = 24.$$

$$8 - 16 + k = 24, k = 32. \text{ Pass back: } 2\sqrt{DX} = 2\sqrt{32(32)} = 64. \quad \mathbf{A = 32, Pass 64}$$

Seat E Pink Relay

$$\begin{bmatrix} x & 1 & 1 \\ 2 & 2 & 3 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 \\ 2 & 3 \\ 2 & -2 \end{bmatrix} = \begin{bmatrix} x+2+2 & x+3-2 \\ 2+4+6 & 4+6-6 \end{bmatrix} = \begin{bmatrix} \frac{x}{2}+4 & x+1 \\ 12 & 4 \end{bmatrix} = -16, 2x + 16 - 12x - 12 = -16 \rightarrow$$

$$-10x = -20, x = 2. \text{ Pass in: } 7\sqrt[3]{X} = 7\sqrt[3]{64} = 56. \quad \mathbf{A = 2, Pass 56}$$

Seat A Yellow Relay

$$\sqrt{245} = 7\sqrt{5}. 5\sqrt{7} = \sqrt{c}, \text{ so } c = 175. \text{ Pass back: } A - 15 = 160. \quad \mathbf{A = 175, Pass 160}$$

Seat B Yellow Relay

$$\frac{x+5}{47-x-2} = \frac{2}{3} \rightarrow 3x + 15 = 90 - 2x \rightarrow 5x = 75, x = 15. \text{ Orig. frac.} = \frac{15}{32} = B.$$

$$\text{Pass back: } BX = \frac{15}{32}(160) = 75. \quad \mathbf{B = 15/32, Pass 75}$$

Seat C Yellow Relay

Since $BD = 6\sqrt{3}$, then $DC = 6$ and $BC = 12$. Thus perimeter = 36. Pass back:

$$\left(\frac{X}{\sqrt{C+9}}\right)^2 = \left(\frac{75}{\sqrt{36+9}}\right)^2 = \left(\frac{75}{15}\right)^2 = 5^2 = 25.$$

C = 36, Pass 25

Seat D Yellow Relay

Same as Pink C: $\frac{-b}{2a} = \frac{-8}{2(2)} = -2$. $2(-2)^2 + 8(-2) + k = -8 \rightarrow 8 - 16 + k = -8, k = 0 = D$.

Pass back: $D + X + 2 = 0 + 25 + 2 = 27$.

D = 0, Pass 27

Seat E Yellow Relay

$$\begin{bmatrix} \frac{x}{2} & 1 & 3 \\ -3 & 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 & 1 \\ 3 & 2 \\ -2 & 2 \end{bmatrix} = \begin{bmatrix} x+3-6 & \frac{x}{2}+2+6 \\ -6+6-2 & -3+4+2 \end{bmatrix} = \begin{bmatrix} x-3 & \frac{x}{2}+8 \\ -2 & 3 \end{bmatrix} = 23 \rightarrow 3x-9 + x+16 = 23 \rightarrow$$

$$4x + 7 = 23 \rightarrow 4x = 16, x = 4.$$

$$\text{Pass in: } \left(\frac{X}{E-1}\right)^4 = \left(\frac{27}{4-1}\right)^4 = 9^4 = 6561.$$

E = 4, Pass 6561

Answer Sheet – States 2019

Individuals Round 1 Individuals Round 2 Individuals Round 3

- | | | |
|---------------------------|-------------------------------------|--------|
| 1. $6x^3 - 2x^2 + 3x - 2$ | 1. -2010 | 1. 100 |
| 2. -5 | 2. 57 | 2. -40 |
| 3. 2025 | 3. $25/4$ or $6\frac{1}{4}$ or 6.25 | 3. 105 |

Individuals Round 4 Individuals Round 5 Individuals Round 6

- | | | |
|----------------|---|-------|
| 1. subtract, 5 | 1. -93 | 1. 28 |
| 2. 24 | 2. 33, 47 | 2. 3 |
| 3. 7 | 3. $121\frac{1}{9}$ or $\frac{1090}{9}$ | 3. 3 |

Team Round 1

- | | |
|--------|-------|
| 1. 6 | 4. 8 |
| 2. -28 | 5. 21 |
| 3. 30 | 6. 6 |

Team Round 2

- | | | |
|------------------------|------------------------|-----------------|
| 1. $-32b^2 - 11b + 41$ | 4. $1/2$ | 7. $48\sqrt{3}$ |
| 2. 16:3 or 16/3 | 5. $\frac{1000\pi}{3}$ | 8. 4 |
| 3. -6 | 6. 16 | |

Blue Relay

- A = 4, Pass 32
B = 3, Pass 37
C = 74, Pass 8
D = 144, Pass 128
E = 125, Pass 9

Green Relay

- A = 8, Pass 105
B = 15, Pass 7
C = 18, Pass 25,000
D = 27,000, Pass 5
E = 9, Pass 625

Pink Relay

- A = 275, Pass 256
B = 11/16, Pass 110
C = 108, Pass 32
D = 32, Pass 64
E = 2, Pass 56

Yellow Relay

- A = 175, Pass 160
B = 15/32, Pass 75
C = 36, Pass 25
D = 0, Pass 27
E = 4, Pass 6561