Name: Key

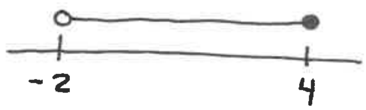
AB Calculus – Summer Assignment
2017 - 2018

Refer to the Review & Instructional Packet for examples.

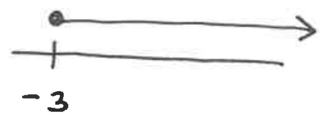
I. Interval Notation/ Solving Inequalities

1 – 4. Write each set using interval notation and sketch a quick graph.
[See Review & Instructional Packet – p. 1 "Interval Notation"]

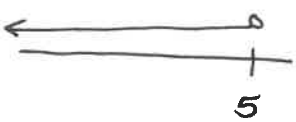
1. $-2 < x \le 4$ $(-2, 4]$



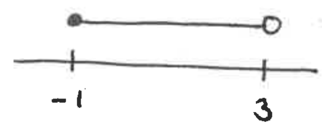
2. $x \ge -3$ $[-3, \infty)$



3. $x < 5$ $(-\infty, 5)$



4. $3 > x \ge -1$ $[-1, 3)$

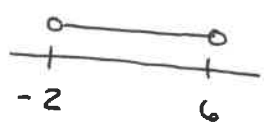


5 – 10. Solve the inequality, express your answer in interval notation, and sketch the solution.

5. $x + 3 > 1$ *and* $x - 2 < 4$

$x > -2$ & $x < 6$

$(-2, 6)$



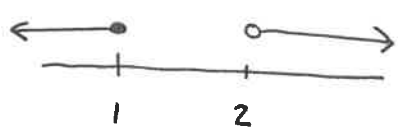
6. $3x - 2 \le 1$ *or* $4x + 1 > 9$

$3x - 2 \le 1$ $4x + 1 > 9$

$3x \le 3$ $4x > 8$

$x \le 1$ or $x > 2$

$(-\infty, 1] \cup (2, \infty)$



Great OR Less ThAND

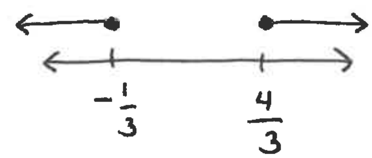
7 – 10. Solve the inequality, express your answer in interval notation, and sketch the solution.

7. $|2x-1| \geq \frac{5}{3}$

$2x - 1 \geq \frac{5}{3}$ or $2x - 1 \leq -\frac{5}{3}$

$2x \geq \frac{8}{3}$ $\qquad$ $2x \leq \frac{-2}{3}$

$x \geq \frac{4}{3}$ $\qquad$ $x \leq -\frac{1}{3}$



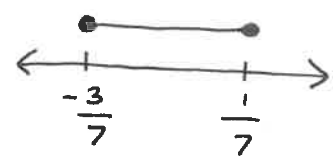
$(-\infty, -\frac{1}{3}] \cup [\frac{4}{3}, \infty)$

8. $|7t+1| \leq 2$

$7t + 1 \leq 2$ and $7t + 1 \geq -2$

$7t \leq 1$ $\qquad$ $7t \geq -3$

$t \leq \frac{1}{7}$ $\qquad$ $t \geq -\frac{3}{7}$

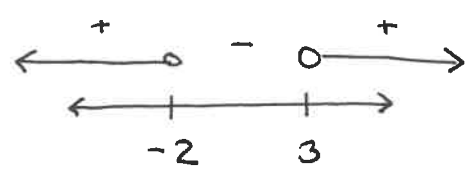


$[-\frac{3}{7}, \frac{1}{7}]$

9. $x^2 - x - 6 > 0$ $\qquad$ [See Review & Instructional Packet p. 2 Polynomial & Rational Inequalities.]

$(x-3)(x+2) = 0$

$x = 3$ $\qquad$ $x = -2$



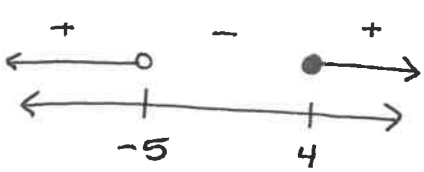
$(-\infty, -2) \cup (3, \infty)$

10. $\frac{x-4}{x+5} \geq 0$ $\qquad$ [See Review & Instructional Packet p. 3-4 Rational Inequalities Ex. 3]

critical points

$x \neq -5$

$x = 4$



$(-\infty, -5) \cup [4, \infty)$

II. Fractional Exponents/Logarithms/Log Properties

Fractional Exponents

$$x^{\frac{a}{b}} = \sqrt[b]{x^a} = \left(\sqrt[b]{x}\right)^a$$

Examples:

1. $32^{\frac{3}{5}} = \left(\sqrt[5]{32}\right)^3 = 2^3 = 8$

2. $125^{-\frac{2}{3}} = \dfrac{1}{125^{\frac{2}{3}}} = \left(\dfrac{1}{\sqrt[3]{125}}\right)^2 = \left(\dfrac{1}{5}\right)^2 = \dfrac{1}{25}$

1 – 4. Simplify completely.

1. $27^{\frac{2}{3}} = \sqrt[3]{27}^{\,2}$
$= 3^2$
$= \boxed{9}$

2. $16^{\frac{3}{4}} = \sqrt[4]{16}^{\,2}$
$= 2^2$
$= \boxed{4}$

3. $64^{-\frac{1}{3}} = \dfrac{1}{\sqrt[3]{64}}$
$= \boxed{\dfrac{1}{4}}$

4. $81^{-\frac{3}{4}} = \dfrac{1}{\sqrt[4]{81}^{\,3}}$
$= \dfrac{1}{3^3} = \boxed{\dfrac{1}{27}}$

5 – 10. Solve for x. [See Review & Instructional Packet p. 5 Logarithms.]

5. $\log_2 32 = x$
$2^x = 32$
$2^x = 2^5$
$\boxed{x = 5}$

6. $\log_3 \dfrac{1}{27} = x$
$3^x = \dfrac{1}{27}$
$3^x = 3^{-3}$
$\boxed{x = -3}$

7. $\log_5 x = 3$
$5^3 = x$
$\boxed{125 = x}$

8. $\log_7 \dfrac{1}{49} = x$
$7^x = \dfrac{1}{49}$
$7^x = 7^{-2}$
$\boxed{x = -2}$

9. $\log_3 x = -2$
$3^{-2} = x$
$\boxed{\dfrac{1}{9} = x}$

10. $\log_{32} x = \dfrac{2}{5}$
$32^{2/5} = x$
$\sqrt[5]{32}^{\,2} = x$
$2^2 = x$
$\boxed{4 = x}$

11 – 12. Use logarithmic identities to write each expression as a single logarithm.
[See Review & Instructional Packet p. 6 Simplifying with Logarithmic Identities]

11. $2\ln 4 - \ln 6 + 3\ln 2 =$

$$= \ln \frac{4^2 (2^3)}{6}$$

$$= \ln \frac{(16)(8)}{6}$$

$$= \boxed{\ln 64/3}$$

12. $2\ln(x+3) - \ln(x^2 + 5x + 6) =$

$$= \ln \frac{(x+3)^2}{x^2 + 5x + 6}$$

$$= \ln \frac{(x+3)^2}{(x+3)(x+2)}$$

$$= \boxed{\ln \frac{x+3}{x+2}}$$

13. Solve the logarithmic equation.

$\log_2(1-3x) = -1$

$$2^{-1} = 1 - 3x$$

$$\frac{1}{2} = 1 - 3x$$

$$-\frac{1}{2} = -3x$$

$$\boxed{\frac{1}{6} = x}$$

III. Piecewise Defined Functions

Evaluate the piecewise-defined function as indicated, and then sketch its graph.
[See Review & Instructional Packet p. 7 Piecewise-Defined Functions.]

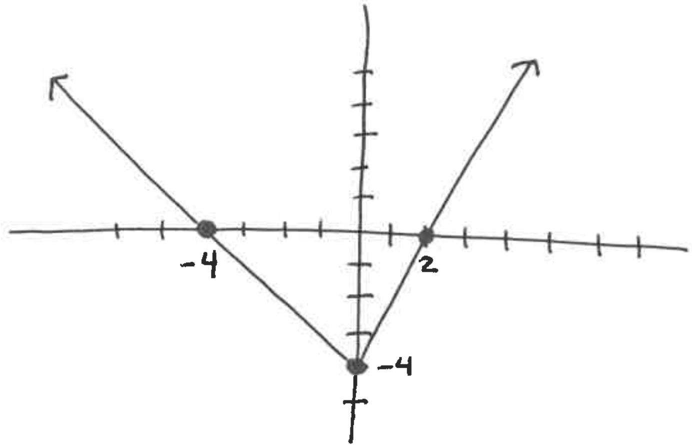
1. $f(x) = \begin{cases} -x-4, & x \le 0 \\ 2x-4, & x > 0 \end{cases}$

$f(-2) = -(-2) - 4 = \boxed{-2}$

$f(0) = -(0) - 4 = \boxed{-4}$

$f(3) = 2(3) - 4$

$= \boxed{2}$

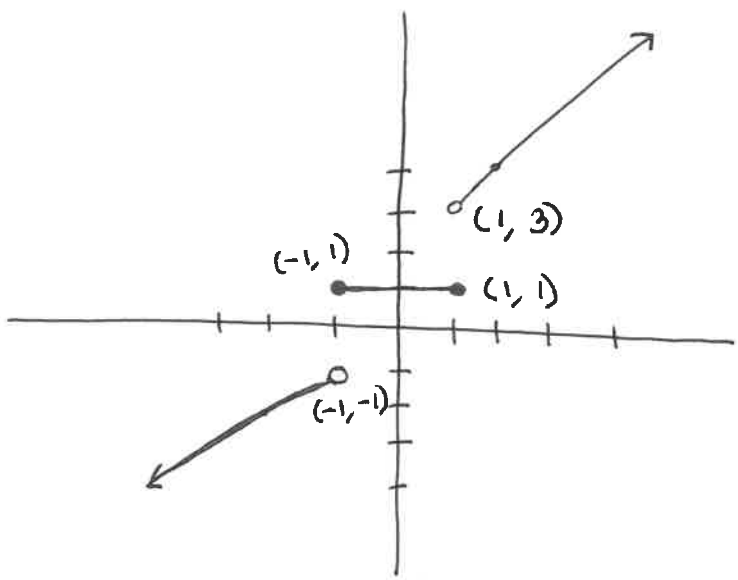


2. $f(x) = \begin{cases} x, & x < -1 \\ 1, & -1 \le x \le 1 \\ x+2, & x > 1 \end{cases}$

$f(-3) = -3$

$f(-1) = 1$

$f(4) = 6$



IV. Area of a Trapezoid

The formula for the area of a trapezoid is $A = \frac{h}{2}\cdot(b_1 + b_2)$ A represents the area, h represents the measure of the altitude, and b_1 and b_2 represent the measures of the bases.

Example 1. Find the area of a trapezoid with bases 12 cm and 7 cm long and an altitude of length 5 cm.

Solution: $A = \frac{1}{2}h(b_1 + b_2) = \frac{5}{2}(12+7) = \frac{95}{2}$ or 47.5 cm

Example 2. A trapezoid of area 150 has one base of length 18 and an altitude of length 12. Find the length of the other base.

Solution:

$$A = \frac{1}{2}h(b_1 + b_2)$$

$$150 = \frac{1}{2}\cdot 12(18 + x)$$

$$150 = 108 + 6x$$

$$42 = 6x$$

$$7 = x$$

1 – 4. Find the area of each trapezoid given the following values.

1. $h = 6$, $b_1 = 22$, $b_2 = 17$

$A = \frac{1}{2}(6)(22+17)$

$= \boxed{117}$

2. $b_2 = 6.25$, $b_1 = 12.5$, $h = 4$

$A = \frac{1}{2}(4)(12.5 + 6.25)$

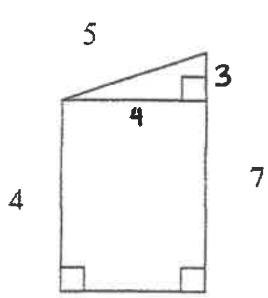
$= \boxed{37.5}$

3. $h = \frac{5}{8}, b_1 = \frac{3}{4}, b_2 = \frac{1}{2}$

$A = \frac{1}{2}\left(\frac{5}{8}\right)\left(\frac{3}{4} + \frac{1}{2}\right)$

$= \boxed{\frac{25}{64}}$

4.



$A = \frac{1}{2}(4)(4+7)$

$= \boxed{22}$

V. Composition of Functions.

Let $f(x) = 3x + 1$ and $g(x) = x^2 - x$.

1. $g(-3) = (-3)^2 - (-3)$

   $= 9 + 3$

   $= \boxed{12}$

2. $f(g(x)) = 3(x^2 - x) + 1$

   $\boxed{= 3x^2 - 3x + 1}$

3. $g(f(x)) = (3x+1)^2 - (3x-1)$

   $= 9x^2 + 6x + 1 - 3x + 1$

   $\boxed{= 9x^2 + 3x}$

4. $f(g(x^3)) = 3(x^3)^2 - 3(x^3) + 1$

   $\boxed{= 3x^6 - 3x^3 + 1}$

VI. Equation of a line:

Slope Intercept form: $y = mx + b$

Vertical Line (slope is undefined): $x = c$

Point-slope form: $y - y_1 = m(x - x_1)$

Horizontal line (slope is 0): $y = c$

1. Write the equation of a line passing through (3,-4) with an undefined slope.

   $x = 3$

2. Use the point-slope form to write the equation of a line passing through (-2,3) with slope $\frac{2}{3}$.

   $y - 3 = \frac{2}{3}(x + 2)$

   $y = \frac{2}{3}x + \frac{13}{3}$

3. Write the equation of a line passing through (-4,5) with a slope of 0.

   $y = 5$

4. Write the equation of a line with an x-intercept $(-3,0)$ and a y-intercept $(0,4)$.

   $4 - 0 = m(0 + 3)$

   $\frac{4}{3} = m$

   $y = \frac{4}{3}x + 4$

VII. Domain

State the domain of the following functions in interval notation:
[See Review & Instructional Packet p. 8 Domain.]

1. $y = \frac{2x+3}{4x-2}$ $x \neq \frac{1}{2}$

$\boxed{(-\infty, 1/2) \cup (1/2, \infty)}$

2. $y = \frac{5x}{x^2-5x+6}$ $(x-3)(x-2) \neq 0$

$x \neq 3, 2$

$\boxed{(-\infty, 2) \cup (2, 3) \cup (3, \infty)}$

3. $y = \sqrt{x+3}$

$x+3 \geq 0$

$x \geq -3$

$\boxed{[-3, \infty)}$

4. $y = \frac{\sqrt{2x-5}}{3x-10}$ $2x-5 \geq 0$ $x \neq \frac{10}{3}$

$x \geq 5/2$

$\boxed{[5/2, 10/3) \cup (10/3, \infty)}$

VIII. Horizontal and Vertical Asymptotes

[See Review & Instructional Packet p. 9 Asymptotes.]

Determine all horizontal and vertical asymptotes:

1. $f(x) = \frac{4}{x^2}$

HA: $y = 0$

VA: $x = 0$

2. $f(x) = \frac{3x^2}{x^2-4}$

HA: $y = 3$

VA: $x = \pm 2$

3. $f(x) = \frac{3x+2}{x^2+x-2} = \frac{3x+2}{(x+2)(x-1)}$

HA: $y = 0$

VA: $x = -2$, $x = 1$

4. $y = \frac{2x^3-7}{3x^2-6x} = \frac{2x^3-7}{3x(x-2)}$

HA: slant assymptote
no horizontal

VA: $x = 0$, $x = 2$

5. $y = \frac{x^2-x-6}{x^2-2x-8} = \frac{(x-3)(x+2)}{(x-4)(x+2)}$

HA: $y = 1$

VA: $x = 4$

removable discontinuity
at $x = -2$

6. $y = \frac{5x^2+10x}{3x-6x^2} = \frac{5x^2+10x}{-3x(2x-1)}$

HA: $y = -\frac{5}{6}$

VA: $x = 0$, $x = \frac{1}{2}$

Domain/Rational Functions **EXERCISES 4.5**

EXERCISES 1–8 ~~*Find the domain and zeros of the rational*~~ *function and match it with its graph.*

1. $f(x) = \frac{1}{x-2}$ $y=0$ $x=2$ no x-int

2. $f(x) = \frac{x}{x-1}$ $y=1$ $x=1$ $x=0$ x-int

3. $f(x) = \frac{x+1}{x}$ $y=1$ $x=0$ $x=-1$ x-int

4. $f(x) = \frac{x-2}{x-1}$ $y=1$ $x=1$ $x=2$ x-int

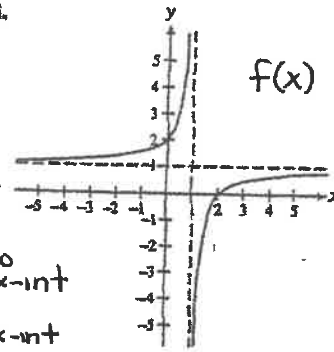
5. $f(x) = \frac{1}{x+2}$ $y=0$ $x=-2$ no x-int

6. $f(x) = \frac{1}{x^2+x-2}$ $y=0$ $x=-2, 1$ no x-int

7. $f(x) = \frac{x^2-4}{x^2-1}$ $y=1$ $x=\pm 1$ $x=\pm 2$ x-int

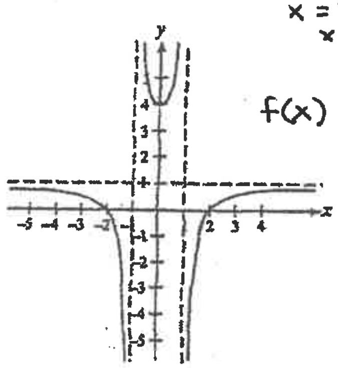
8. $f(x) = \frac{x+1}{x+2}$ $y=1$ $x=-2$ $x=-1$ x-int

i.



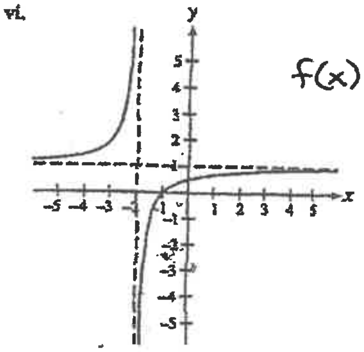
$f(x) = \frac{x-2}{x-1}$ #4

ii.



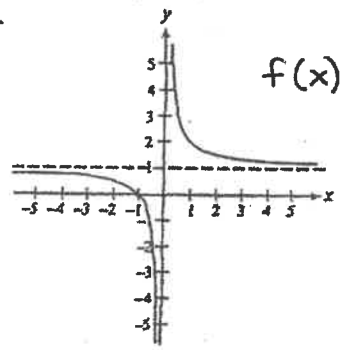
$f(x) = \frac{x^2-4}{x^2-1}$ #7

vi.



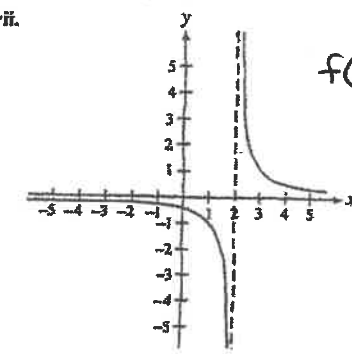
$f(x) = \frac{x+1}{x+2}$ #8

iii.



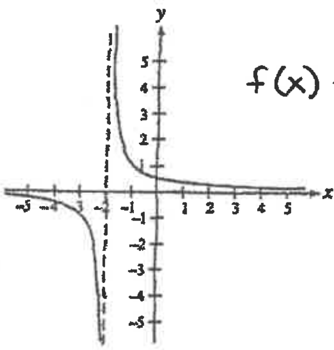
$f(x) = \frac{x+1}{x}$ #3

vii.



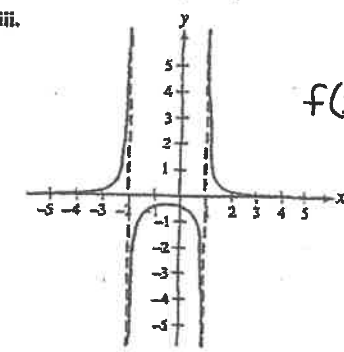
$f(x) = \frac{1}{x-2}$ #1

iv.



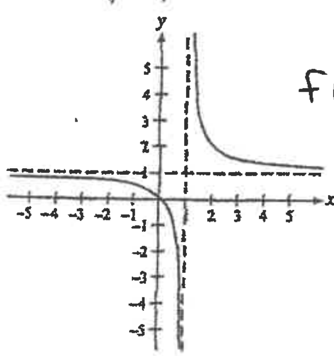
$f(x) = \frac{1}{x+2}$ #5

viii.



$f(x) = \frac{1}{x^2+x-2}$ #6

v.



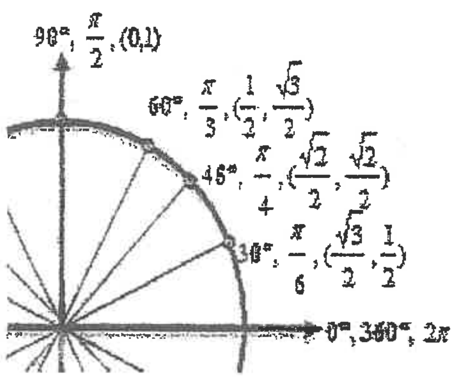
$f(x) = \frac{x}{x-1}$ #2

IX. Trig Values

Convert to radians:

1. $45° =$ $\pi/4$
2. $60° =$ $\pi/3$
3. $90° =$ $\pi/2$
4. $30° =$ $\pi/6$

UNIT CIRCLE

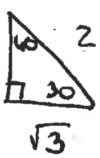


You can determine the sine or the cosine of any standard angle on the unit circle. The x-coordinate of the circle is the cosine and the y-coordinate is the sine of the angle. Recall tangent is defined as sin/cos or the slope of the line.

Examples:

$\sin\frac{\pi}{2} = 1$ $\cos\frac{\pi}{2} = 0$ $\tan\frac{\pi}{2} = und$

***You must have these memorized OR know how to calculate their values without the use of a calculator.**



a.) $\sin\pi$ 0

b.) $\cos\frac{3\pi}{2}$ 0

c.) $\sin\left(-\frac{\pi}{2}\right)$ 1

S | A
T | C

d.) $\sin\left(\frac{5\pi}{4}\right)$ $-\frac{\sqrt{2}}{2}$

e.) $\cos\frac{\pi}{4}$ $\frac{\sqrt{2}}{2}$

f.) $\cos(-\pi)$ -1

g) $\cos\frac{\pi}{3}$ $\frac{1}{2}$

S +

h) $\sin\frac{5\pi}{6}$ $\frac{1}{2}$

S | A

i) $\cos\frac{2\pi}{3}$ $-\frac{1}{2}$

j) $\tan\frac{\pi}{4}$ 1

k) $\tan\pi$ 0

l) $\tan\frac{\pi}{3}$ $\sqrt{3}$

Find the values of the inverse trig functions – give your answer in radians.

1. $Sin^{-1}\frac{1}{2} =$ $\frac{\pi}{6}$
2. $Cos^{-1}\frac{\sqrt{3}}{2} =$ $\frac{\pi}{6}$
3. $Tan^{-1}1 =$ $\frac{\pi}{4}$
4. $Sin^{-1}\frac{\sqrt{2}}{2} =$ $\frac{\pi}{4}$

10

LESSON 4.23

Read each sentence and decide which of the graphs below matches the situation.

d 1. As Emily rides round and round on a Ferris wheel, her height off the ground is a function of time.

e 2. As the price goes up, the demand for the product goes down. Price is a function of demand. Assume there may be a price so high no one would pay it.

h 3. A cup of hot coffee cools to room temperature. Temperature is a function of time.

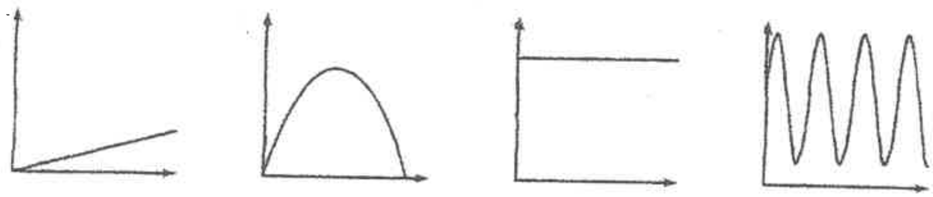
a 4. The more Fred works, the more money he earns. Wages are a function of hours worked.

b 5. A firecracker is fired into the air. Height is a function of time.

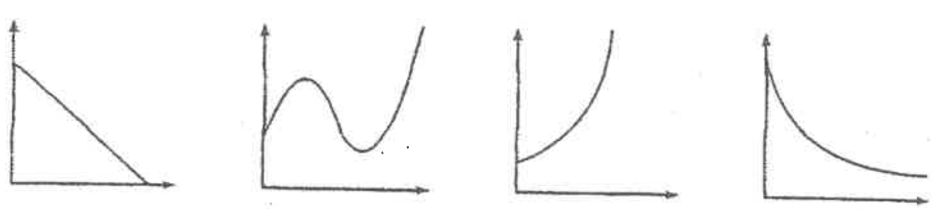
g 6. Bacteria increased at an increasing rate. The number of bacteria is a function of time.

c 7. The temperature held steady at 80° F. Temperature is a function of time.

f 8. The temperature rose, then fell, and then rose again. Temperature is a function of time.



a. b. c. d.

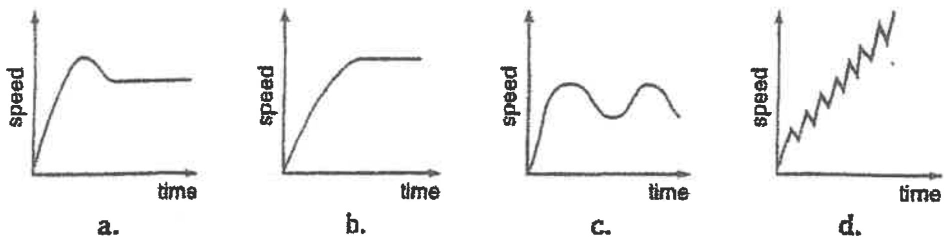


e. f. g. h.

For each set of graphs, choose the one that best matches the situation:

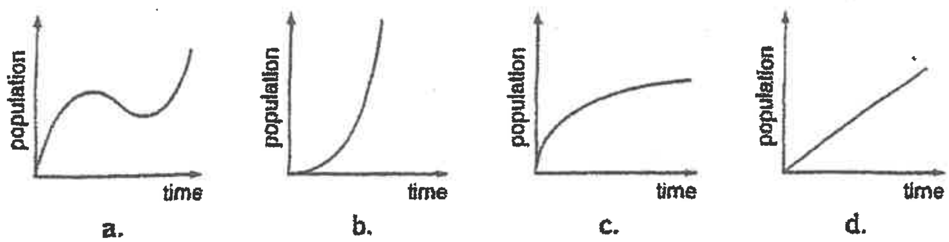
Fumihiko kept increasing his speed until his mother made him slow down and proceed at a constant speed under the limit.

A



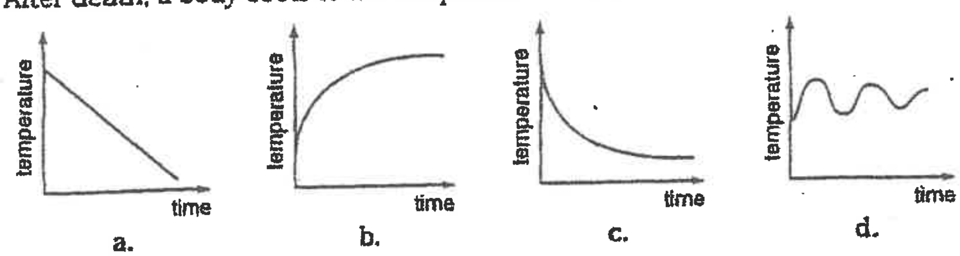
The population of Riverville increased at a rapid rate in the beginning and then leveled off as time passed.

C



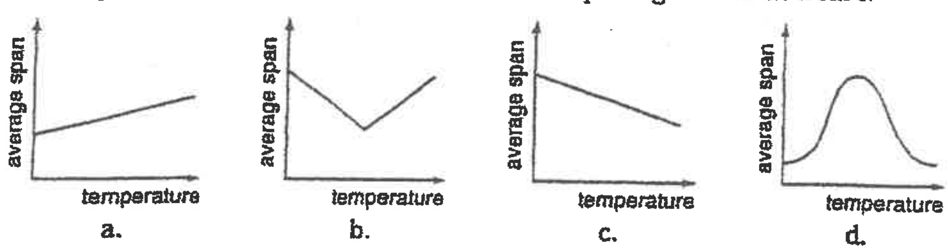
After death, a body cools to the temperature of the room.

C



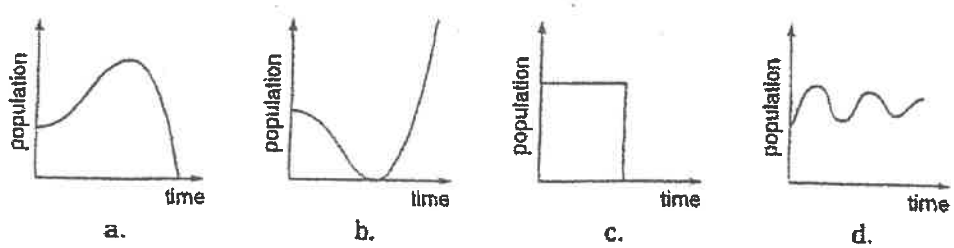
The ideal temperature for a classroom is around 70° F. When the temperature rises above 70°F, the average student's attention span decreases. Similarly, if the temperature falls below 70°F, the attention span again will decrease.

D



In the 1930s in Arizona, the deer population first increased and then decreased until deer became extinct.

A



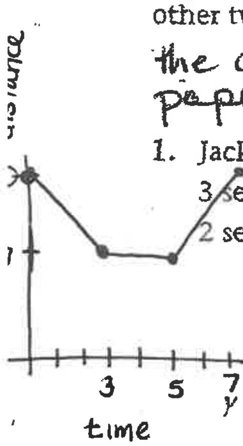
12

Lesson 15

Finding the Right Functions

In Lesson 14, you drew graphs of piecewise functions. Here you will look at those functions and find their equations.

Choose the set of functions that matches each situation. Explain why the other two sets are incorrect. Graph the function that matches the description of each person's motion on separate paper



1. Jack was standing 10 feet from the tree, walked 3 feet toward the tree in 3 seconds, stood for 2 seconds, and then walked back to his original spot in 2 seconds.

Function A

$y = 10 - x$ for $0 \le x \le 3$
$y = 7$ for $3 < x \le 5$
$y = \left(-\frac{1}{2}\right) + \left(\frac{3}{2}\right)x$ for $5 < x \le 7$

Function B

$y = 10 - 3x$ for $0 \le x \le 3$
$y = 1$ for $3 < x \le 5$
$y = 1 + \left(\frac{9}{2}\right)(x - 5)$ for $5 < x \le 7$

Function C

$y = 10$ for $0 \le x \le 3$
$y = 3x + 1$ for $3 < x \le 5$
$y = 31 - 3x$ for $5 < x \le 7$

2. Jamie ran away from the tree at 5 feet per second for 4 seconds, caught her breath for 5 seconds, then went back to the tree moving at 4 feet per second.

Function A

$y = 5x$ for $0 \le x \le 4$
$y = 20$ for $4 < x \le 5$
$y = 20 - 4x$ for $5 < x \le 10$

Function B

$y = 5x$ for $0 \le x \le 4$
$y = 20$ for $4 < x \le 9$
$y = 56 - 4x$ for $9 < x \le 14$

Function C

$y = 5 + 4x$ for $0 \le x \le 4$
$y = 21$ for $4 < x \le 9$
$y = 57 - 4x$ for $9 < x \le 14$

3. Jorge walked away from the tree at 3 feet per second and stopped to wave to a friend after 4 seconds. After another 4 seconds, he continued to walk away at 5 feet per second.

Function A

$y = 3x$ for $0 \le x \le 4$
$y = 12$ for $4 < x \le 8$
$y = 12 + 5x$ for $8 < x$

Function B

$y = 3x$ for $0 \le x \le 4$
$y = 12$ for $4 < x \le 8$
$y = 52 - 5x$ for $8 < x$

Function C

$y = 3x$ for $0 \le x \le 4$
$y = 12$ for $4 < x \le 8$
$y = 5x - 28$ for $8 < x$

4. Look at all the piecewise functions in Questions 1–3. Find the two piecewise functions that are not connected. Can you use these functions to describe a person's motion? Explain.

2A and 3A

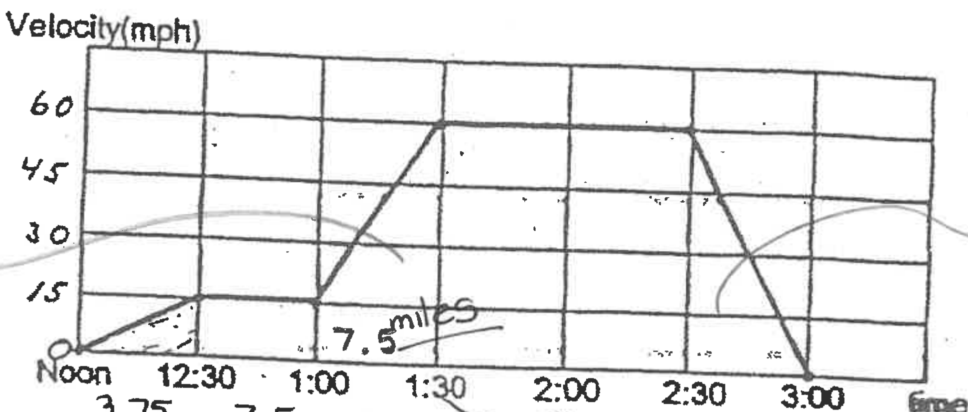
No a person cannot be in 2 different locations at the same moment in time

13

Calculus Networking: Accumulation

HOW FAR DID THE TRAIN TRAVEL?

A passenger train leaves Worcester and travels west. A recording device located on the train records the velocity (mph) at various times during the trip. The velocity-time function is illustrated below in the graph. At 3:00 p.m., how far west is the train from Worcester?



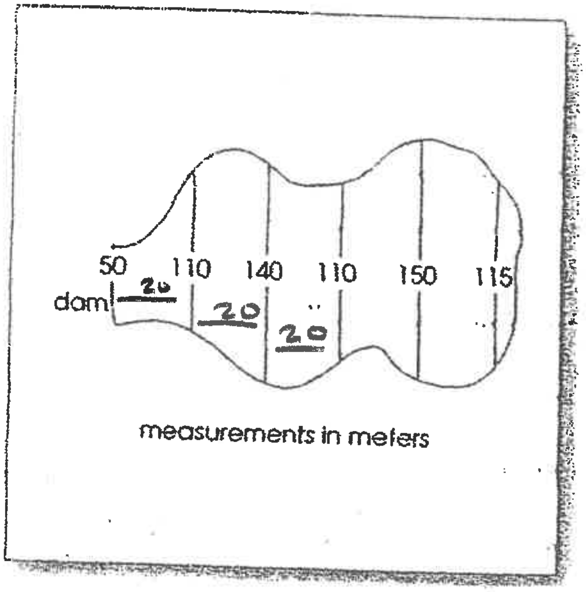
3.75 mi 7.5 mi 18.75 mi 60 mi 15 mi

60 − 15 = 45
1/2 hr
⇒ 22.5
1/2 that
11.25

60 mph
for 1/2 hr
⇒ 30 mi
1/2 that
15 mi

$\boxed{\text{105 miles}}$

1. Lake Surface



To estimate the surface area of a lake, a surveyor takes several measurements, as shown in the figure at left. The surveyor took her measurements every 20 meters, beginning with the lakeside blocked by the dam. Use these measurements to estimate the surface area of the lake. Discuss ways in which you could improve your estimate.

☆Trapezoids

$$A = \frac{1}{2}h(b_1 + b_2)$$

$$A = \frac{1}{2}(20)(50+110) + \frac{1}{2}(20)(110+140) + \frac{1}{2}(20)(140+110)$$
$$+ \frac{1}{2}(20)(110+150) + \frac{1}{2}(20)(150+115)$$

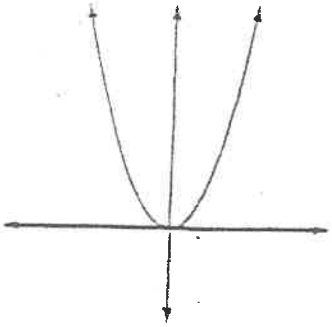
$$= \frac{1}{2}(20)\left[(50+110) + (110+140) + (140+110) + (110+150) + (150+115)\right]$$

$$= 11{,}850 \text{ m}^2$$

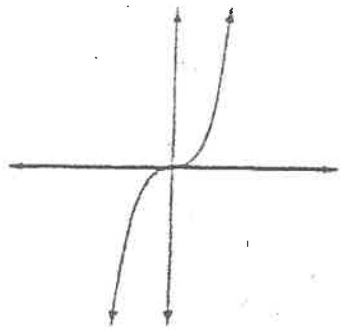
use smaller intervals → more frequent measurements

from: Advanced Placement Program Mathematics Vertical Teams Toolkit, CEEB, 1998

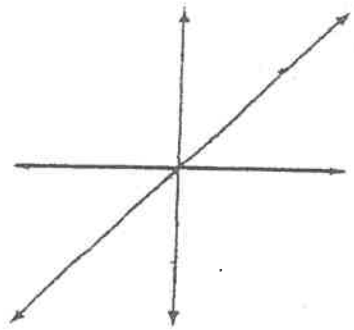
Important Functions to Memorize



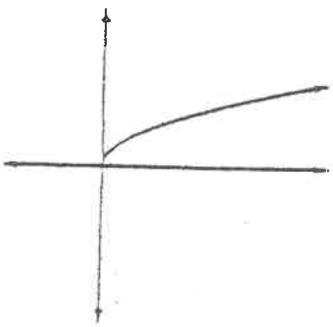
$y = x^2$
Domain: $(-\infty,\infty)$
Range: $[0,\infty)$



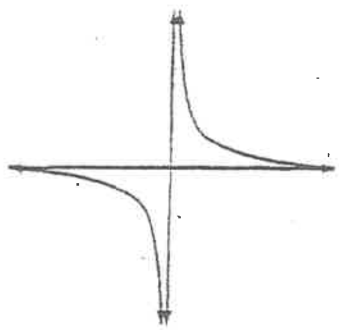
$y = x^3$
Domain: $(-\infty,\infty)$
Range: $(-\infty,\infty)$



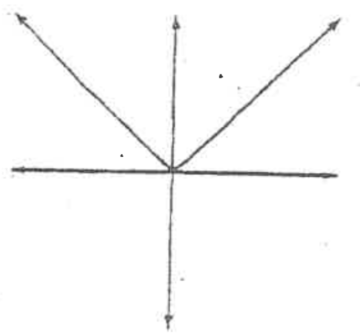
$y = x$
Domain: $(-\infty,\infty)$
Range: $(-\infty,\infty)$



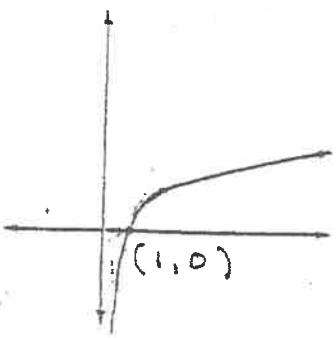
$y = \sqrt{x}$
Domain: $[0,\infty)$
Range: $[0,\infty)$



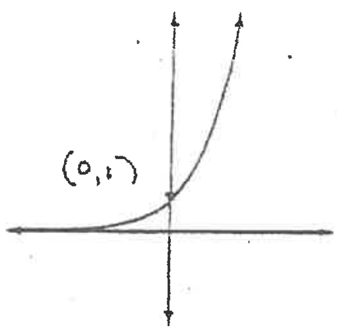
$y = \frac{1}{x}$
Domain: $(-\infty,0)\cup(0,\infty)$
Range: $(-\infty,0)\cup(0,\infty)$



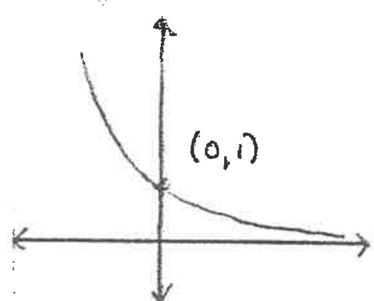
$y = |x|$
Domain: $(-\infty,\infty)$
Range: $[0,\infty)$



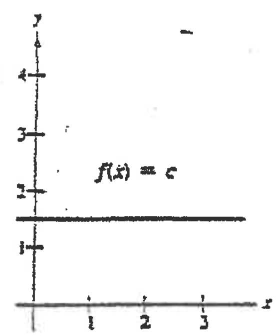
$y = \ln x$
Domain: $(0,\infty)$
Range: $(-\infty,\infty)$
Contains point (1,0)



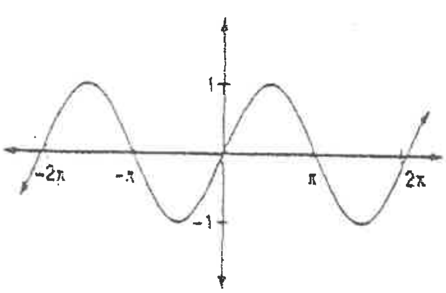
$y = e^x$
Domain: $(-\infty,\infty)$
Range: $(0,\infty)$
Contains point (0,1)



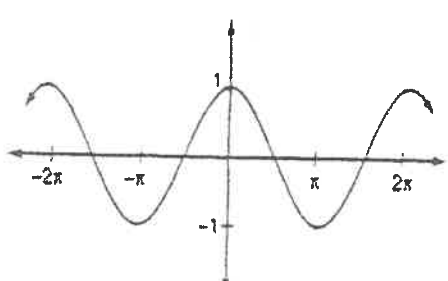
$y = e^{-x}$
Domain: $(-\infty,\infty)$
Range: $(0,\infty)$



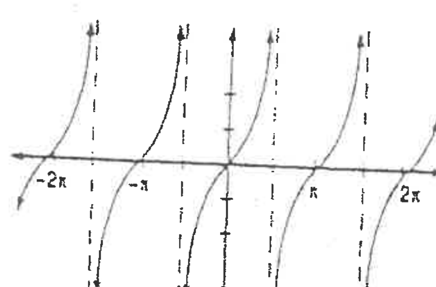
Constant Function



$y = \sin x$



$y = \cos x$



$y = \tan x$